UNIT- I

Balancing of Rotating Masses:-

A Mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite site of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass, is called banlancing of rotating masses.

Important Cases:-

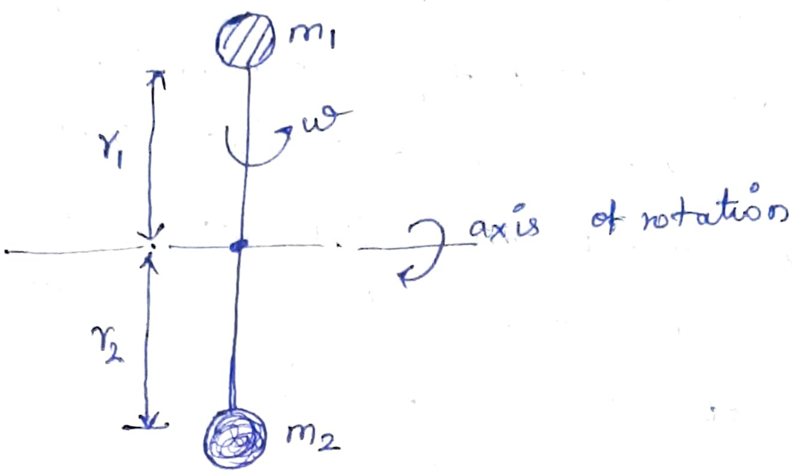
(1) Balancing of a single rotating mass by a single mass rotating in the same plane.

(2) Balancing of a single rotating mass by two masses rotating in different planes.

(3) Balancing of different masses rotating in ~~different~~ the ~~planes~~ same plane.

(4) Balancing of different masses rotating in different planes.

(i) Balancing of a single rotating mass by a single Mass rotating in the same plane



centrifugal force due to first Mass, $F_{c_1} = m_1 \omega_1^2 r_1$

centrifugal force due to second mass, $F_{c_2} = m_2 \omega_2^2 r_2$

Hence $F_{c_1} = F_{c_2}$

$m_1 \omega_1^2 r_1 = m_2 \omega_2^2 r_2$

same speed, $\omega_1 = \omega_2$

$\therefore \boxed{m_1 r_1 = m_2 r_2}$

$m_1 \rightarrow$ Rotating mass.

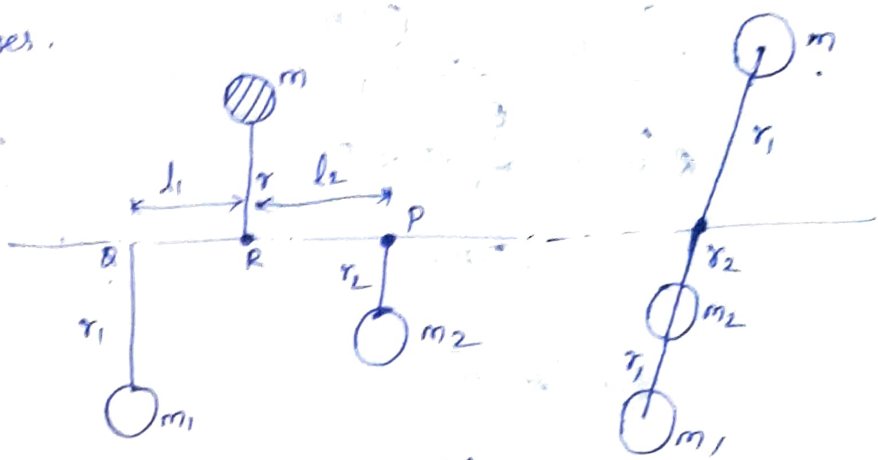
$r_1 \rightarrow$ Radius of rotation of mass m_1

$m_2 \rightarrow$ Balancing Mass.

$r_2 \rightarrow$ Radius of rotation of Mass m_2

(2) Balancing of a single rotating mass By Two Mass rotating in different planes.

(i) when the plane of the ~~distribut~~ disturbing mass lies in b/w the planes of the two balancing masses.



Here, Net dynamic force (or) Net force acting on the shaft is equal to Zero. [

$$\boxed{F_c = F_{c_1} + F_{c_2}}$$

$$m\omega^2 r = m_1\omega_1^2 r_1 + m_2\omega_2^2 r_2$$

Here, $\omega_1 = \omega_2 = \omega$

$$m\cancel{\omega^2} r = \cancel{\omega^2}\left[m_1 r_1 + m_2 r_2\right]$$

$$\boxed{m r = m_1 r_1 + m_2 r_2}$$

Length, $\boxed{l = l_1 + l_2}$

Take Moment about P,

$$F_c \times l_2 = F_{c_1}\left[l_1 + l_2\right].$$

$$F_c l_2 = F_{c_1} l.$$

$$m\cancel{\omega^2} r\, l_2 = m_1\cancel{\omega^2} r_1\, l$$

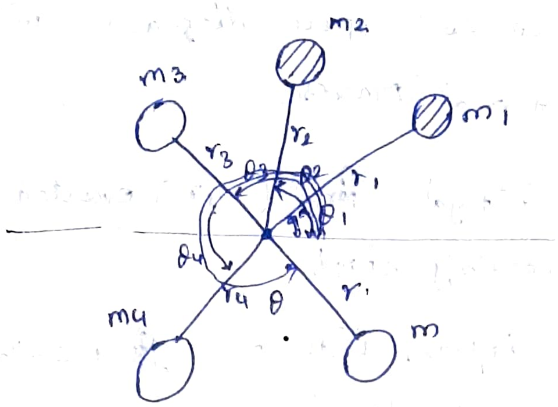
$$\boxed{m_1 r_1 = m r\left[\frac{l_2}{l}\right]}$$

Similarly take moment about Q

$$F_c l_1 = F_{c_2} l$$

$$\boxed{m_2 r_2 = m r\left[\frac{l_1}{l}\right]}$$

(3) Balancing of several Masses Rotating in the same plane



(i) Analytical Method

Sum of Horizontal components.

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$$

Sum of vertical components

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$$

Magnitude of the Resultant force.

$$F_c = \sqrt{\Sigma H^2 + \Sigma V^2}$$

"θ" is angle, which the resultant force makes with the horizontal.

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right].$$

Here $\boxed{F_c = m_\theta r}$

$m \rightarrow$ Balancing Mass.

$r \rightarrow$ Radius of rotation.

∴ The balancing force is then equal to the resultant force, but in opposite direction.

2) Graphical method

i) ~~First of all,~~ Draw the space diagram with the positions of the several Masses.

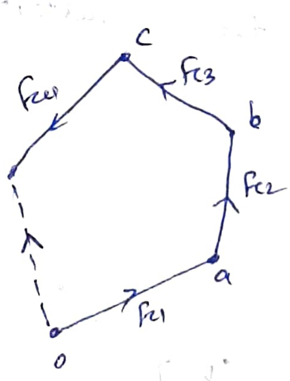
ii) Find out the centrifugal force ($m_1 r_1$) exerted by each mass on the rotating shaft.

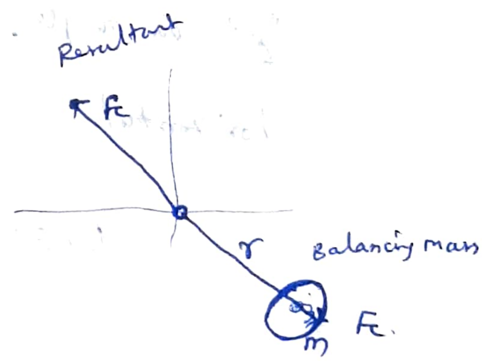
iii) Draw the vector diagram with the ~~given~~ centrifugal force in Magnitude and direction.

iv) As per polygon law of forces, the closing side represents the resultant force in Magnitude and direction.

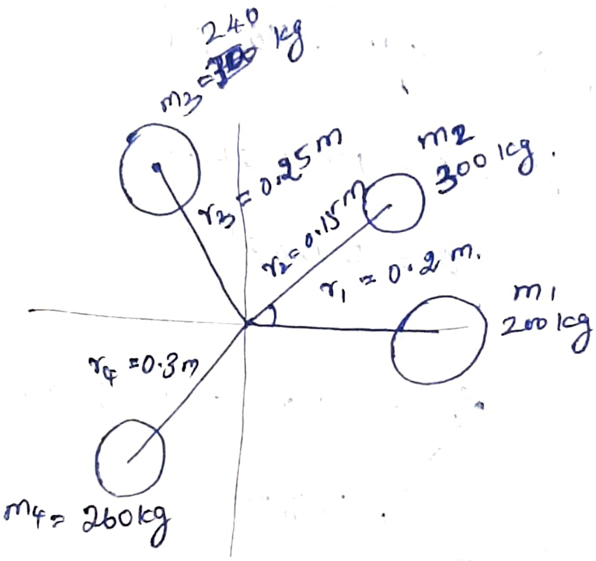
v) The balancing force is, then equal to the resultant force but in opposite direction.

vi) $mr = m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4$, Find out the magnitude of the balancing mass at the given radius.





⊗ Four masses m_1, m_2, m_3 & m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radius of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively. and the angles b/w successive masses are 45°, 75°, and 135°. Find the position and magnitude of the balance mass required, if its radius ~~of~~ rotation is 0.2 m.



$\theta_1 = 0'$
$\theta_2 = 45°$
$\theta_3 = 120'$
$\theta_4 = 255°$.

$m_1 r_1 = 200 \times 0.2 = 40$ kg m.
$m_2 r_2 = 300 \times 0.15 = 45$ kg m
$m_3 r_3 = 240 \times 0.25 = 60$ kg m
$m_4 r_4 = 260 \times 0.3 = 78$ kg m.

Analytical Method

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$$

$$= 40 \cos 0 + 45 \cos 45 + 60 \cos 120' + 78 \cos 255$$

$$= \underline{\underline{21.6 \text{ kg.m}}}$$

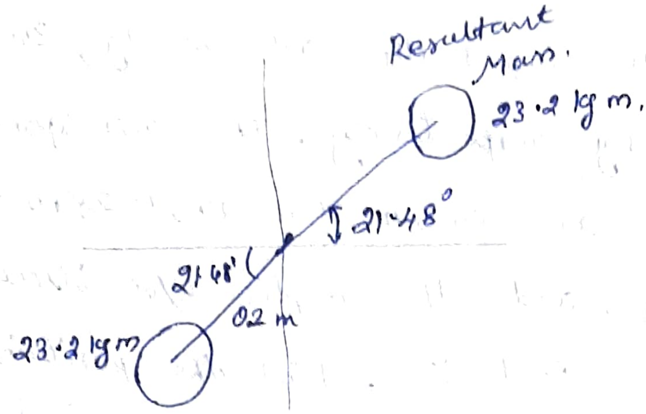
$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$$

$$= 40 \sin 0 + 45 \sin 45 + 60 \sin 120 + 78 \sin 255$$

$$= \underline{\underline{8.5 \text{ kg m}}}$$

Resultant force, $R = \sqrt{\Sigma H^2 + \Sigma V^2} = \underline{\underline{23.2}}$ kg.m

angle, $\theta = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] = \underline{\underline{21.48°}}$



Balancing Mass

$R = mr = 23.2$ kg m.

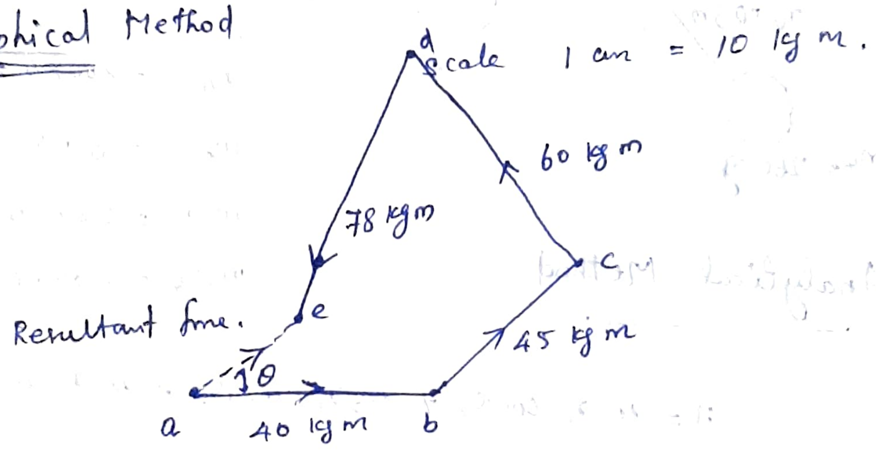
$m[0.2] = 23.2$

Balancing Mass ; $\boxed{m = 116 \text{ kg}}$

angle, $\theta = 180° + 21.48'$

$\boxed{\theta = 201.48°}$

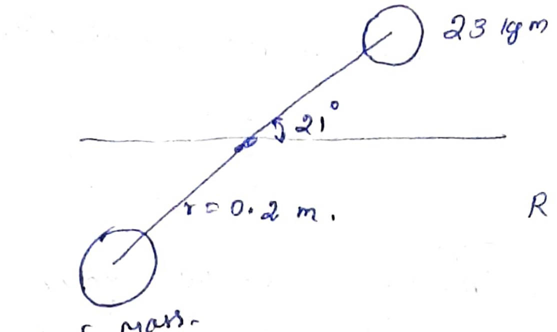
(2) Graphical Method



ae = 2.3 cm, Resultant force, R = 2.3 × 10

= 23 kg m.

$\theta = 21°$



$R = mr = 23$

$\boxed{m = 115 \text{ kg}}$

$\theta = 180 + 21 = 201°$

Flywheel

→ A flywheel used in machines serves as a reservoir, which stores energy during the periode, when the supply of energy is more than the requirement, and releases it during the periode when the requirement of energy is more than the supply.

Coefficient of fluctuation of speed

The difference b/w the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed.

The ratio of the maximum fluctuation of speed to the mean speed is called coeff of fluctuation of speed.

$N_1 \rightarrow$ Max speed

$N_2 \rightarrow$ Min speed.

Max fluctuation of speed $= N_1 - N_2$

Mean speed $= \frac{N_1 + N_2}{2}$

Coeff of fluctuation of speed $= \frac{(N_1 - N_2) 2}{N_1 + N_2} = \frac{N_1 - N_2}{N}$

C_s

$= \frac{\omega_1 - \omega_2}{\omega}$

Coefficient of steadiness

The reciprocal of the coefficient of fluctuation of speed is known as Coeff of steadiness.

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2} = \frac{\omega}{\omega_1 - \omega_2}$$

Energy stored in flywheel

Mean kinetic energy of the flywheel

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 r^2 = \frac{1}{2} I \omega^2 = \frac{1}{2} m k^2 \omega^2$$

Nm or Joule.

Fluctuation of Energy, $\Delta E = \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$

$$= \frac{1}{2} I \left[\omega_1^2 - \omega_2^2\right]$$

$$= \frac{1}{2} I \left[\omega_1 + \omega_2\right]\left[\omega_1 - \omega_2\right]$$

$$= \frac{1}{2} I \left[\omega_1 + \omega_2\right]\left[\frac{\omega_1 - \omega_2}{\omega}\right] \times \omega$$

$$= \cancel{\tfrac{1}{2}2}\, I \omega^2 C_s$$

$$\boxed{\Delta E = 2 E C_s}$$ in Nm or Joule.

The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration (k) is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kNm. If the mean speed of the engine is 120 rpm. Find the max. and Min speeds.

$m = 6.5$ tonnes $= 6500$ kg, $k = 1.8$ m, $\Delta E = 56$ kNm $= 56 \times 10^3$ Nm

$\frac{N_1 + N_2}{2} = 120$ rpm

$$\Delta E = I \omega^2 C_s$$

$$56 \times 10^3 = m k^2 \times \left[\frac{2 \pi N}{60}\right]^2 \times \left[\frac{N_1 - N_2}{N}\right]$$

$$56 \times 10^3 = 6500 \times 1.8^2 \times \left[\frac{2 \times \pi \times 120}{60}\right]^2 \times \frac{(N_1 - N_2)}{120}$$

C_s

$$\boxed{N_1 - N_2 = 2 \text{ rpm}}$$

$$N = \frac{N_1 + N_2}{2} = 120 \text{ rpm}$$

$$\boxed{N_1 + N_2 = 240 \text{ rpm}}$$

Result

$$\boxed{\begin{aligned} \therefore N_1 &= 121 \text{ rpm} \\ N_2 &= 119 \text{ rpm} \end{aligned}}$$

(2) A horizontal Cross Compound steam engine developes 300 kW at 90 rpm. The coeff of fluctuation of energy as found from the turning moment diagram is to be 0.1 and the fluctuation of speed is to be kept within ±0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2 m.

$p = 300\ kW = 300 \times 10^3$ watt, $N =$ rpm, $C_E = 0.1$,

$K = 2\ m.$

$$\omega = \frac{2\pi N}{60} = \underline{9.426 \text{ rad/sec}}$$

$$C_s = \frac{\omega_1 - \omega_2}{\omega} = \frac{(\omega + 0.005\omega) - (\omega - 0.005\omega)}{\omega}$$

$$= \frac{\cancel{\omega} + 0.5\omega - \cancel{\omega} + 0.5\omega}{\omega} = \frac{0.01\cancel{\omega}}{\cancel{\omega}}$$

$$\boxed{C_s = 0.01}$$

power, $P = \dfrac{2\pi N T}{60} =$

1) The turning moment diagram for a multicylinder engine has been drawn to a scale 1 mm = 600 NM vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows :-

$+52, -124, +92, -140, +85, -72$ & $+107$ mm^2,

when the engine is running at a speed of 600 rpm. If the total fluctuation of speed is not to exceed $\pm 1.5\%$ of the mean. Find the necessary mass of the flywheel of radius 0.5 m.

$N = 600$ rpm (or) $\omega = \frac{2\pi \times 600}{60} = 62.84$ rad/sec, $r = 0.5$ m.

$\omega_1 - \omega_2 = 3\% \omega \Rightarrow \boxed{\omega_1 - \omega_2 = 0.03\,\omega}$

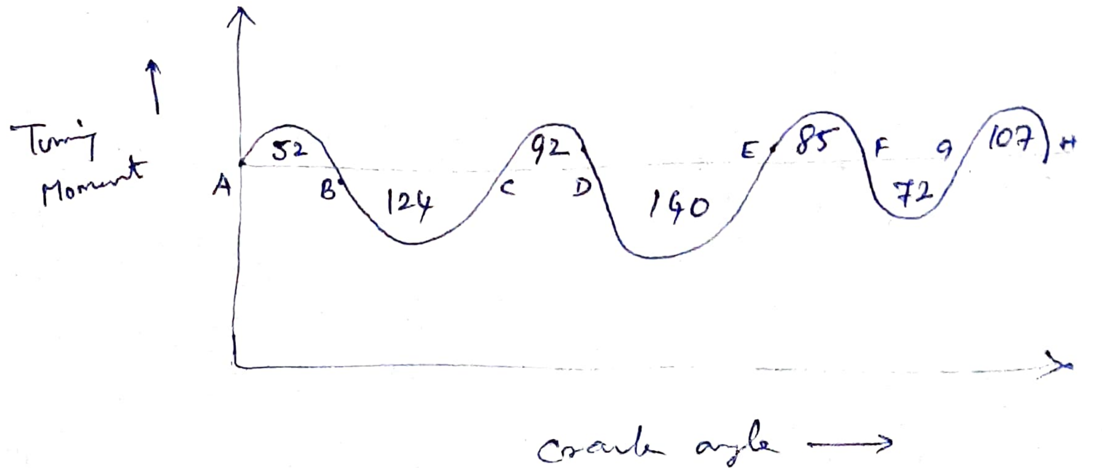
$C_s = \frac{\omega_1 - \omega_2}{\omega} = \underline{0.03}$

1 mm = 600 Nm

1 mm = $3° = 3 \times \frac{\pi}{180} = \frac{\pi}{60}$ rad,

$1\ mm^2 = 600 \times \frac{\pi}{60}$

$= \underline{31.42\ Nm}$



Energy at A = E

B = E+52 ⟶ Max Energy,

C = E+52−124 = E−72

D = E−72+92 = E+20

E = E+20−140 = E−120 ⟶ Min Energy.

F = E−120+85 = E−35

G = E−35−72 = E−107

H = E−107+107 = E

$\Delta E = \cancel{E}+52 - \cancel{E}+120$

$= 172 \ mm^2$

$= 172 \times 31.42$

$\Delta E = 5404 \ Nm$

$\Delta E = I \omega^2 C_s$

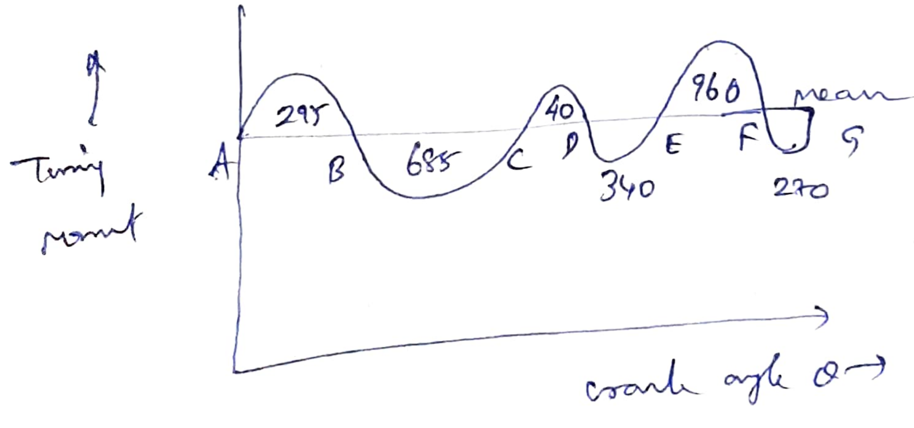
$5404 = m k^2 \omega^2 C_s$

$5404 = m \times 0.5^2 \times 62.84^2 \times 0.03$

m = ~~[illegible]~~ 183 kg

(3) The turning moment diagram for a pertrol engine is drawn to the following scales: Turning moment 1 mm = 5 Nm ; crank angle 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the Mean turming momet line taken in order are 295, 685, 40, 340, 960, 270 mm^2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coeff of fluctuation of speed when the engine runs at 1800 rpm

$m = 36$ kg, $k = 150$ mm $= 0.15$ m, $N = 1800$ rpm, $\omega = 188.52$ rad/sec



Scale 1 mm = 5 Nm

1 mm = 1° = $\frac{\pi}{180}$ rad.

Area → 1 mm^2 = $5 \times \frac{\pi}{180}$ Nm = $\frac{\pi}{36}$ Nm

Energy at A = E

B = E + 295 ⟶ Max energy

C = E+295 − 685 = E − 390

D = E − 390 + 40 = E − 350

E = E − 350 − 340 = E − 690 ⟶ Min Energy.

F = E − 690 + 960 = E + 270

G = E + 270 − 270 = E. (Energy at A)

ΔE = Max energy − Min energy

= $E + 295 - (E - 690) = \cancel{E} + 295 - \cancel{E} + 690$

$\Delta E = 985 \text{ mm}^2$

$\Delta E = 985 \times \frac{\pi}{36} = 86$ Nm (or) Joule

$\therefore \quad \Delta E = m k^2 \omega^2 C_s$

$86 = 36 \times 0.15^2 \times 188.52^2 \times C_s$

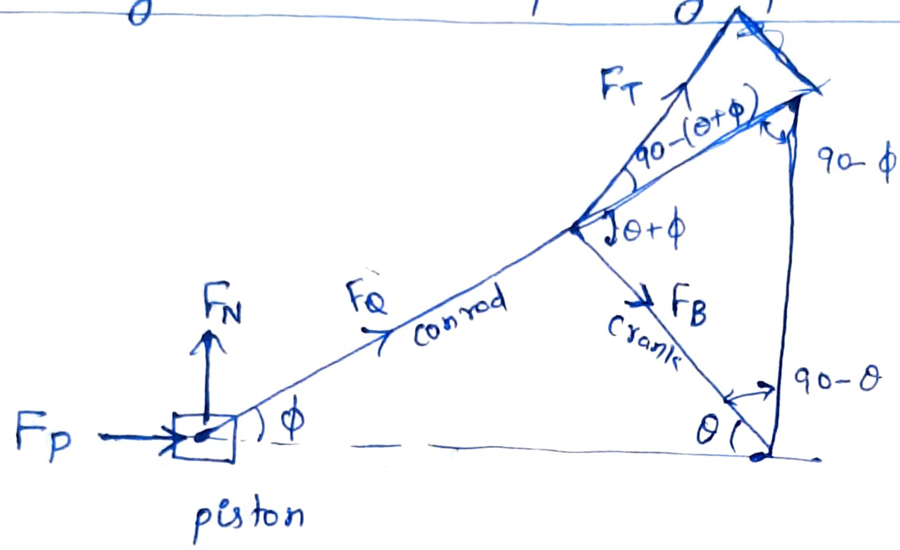
$\boxed{C_s = 0.003}$

Inertia force

→ The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equillibrium position. It is numerically equal to the accelerating force in magnitude, but opposite in direction.

Inertia force = − Accelerating force = $-ma$.

Forces acting on the Reciprocating parts of an Engine:-



Acceleration of the Reciprocating part. (piston)

$$a_R = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Acceleratoring force (or Inertia force of the Reciprocating part.

$$F_I = m_R a_R = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

∴ piston effort, F_p = Net load on the piston ± Inertia force

$$\boxed{F_p = \left[\frac{\pi}{4} D^2\right] p \pm F_I}$$

p → pressure
D → Diameter of piston

Force acting on the Conrod

$$F_{\phi} = F_Q \cos\phi$$

$$\boxed{F_Q = \frac{F_P}{\cos\phi}} \Rightarrow F_Q = \frac{F_P}{\sqrt{1-\frac{\sin^2\theta}{n^2}}}$$

$\sin^2\phi + \cos^2\phi = 1$

$\cos\phi = \sqrt{1-\sin^2\phi}$

$= \sqrt{1-\frac{\sin^2\theta}{n^2}}$

Trust on the sides of the cylinder wall (or) Reaction on the guide bars.

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \times \tan\phi.$$

Crank pin effort

$$F_T = F_Q \sin(\theta+\phi) = \frac{F_P}{\cos\phi} \times \sin(\theta+\phi)$$

$$F_B = F_Q \cos(\theta+\phi) = \frac{F_P}{\cos\phi} \times \cos(\theta+\phi)$$

Crank effort (or) Turning Moment on the crank shaft

$$T = F_T \times r = \frac{F_P}{\cos\phi} \times \sin(\theta+\phi) \cdot r$$

$$= \frac{F_P(\sin\theta\cos\phi + \cos\theta\sin\phi)}{\cos\phi} \times r$$

$$= F_p \left[\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi} \right] r$$

$$= F_p \left[\sin\theta + \cos\theta \tan\phi \right] r. \qquad (n = l/r)$$

$$= F_p \left[\sin\theta + \cos\theta . \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}} \right] r$$

$$l \sin\phi = r \sin\theta.$$

$$\sin\phi = \frac{r}{l} \sin\theta$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$\cos\phi = \sqrt{1 - \sin^2\phi}$$

$$\tan\phi = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}.$$

$$T = F_p . r \left\{ \sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right\}$$

(neglect $\sin^2\theta$)

$$\boxed{T = F_p r \left[\sin\theta + \frac{\sin 2\theta}{2n} \right]}$$

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(1) Find the inertia force for the following data of an IC engine.

Bore = 175 mm, stroke = 200 mm, engine speed = 500 rpm, length of con rod = 400 mm, crank angle = 60° from TDC. and mass of reciprocating part = 180 kg.

Sol

$D = 175$ mm,

Stroke = 200 mm = 0.2 m $\Rightarrow r = \frac{\text{Stroke}}{2} = \underline{0.1\ m}$

$N = 500$ rpm.

$l = 400$ mm

$\theta = 60°$

$m = 180$ kg.

$$n = \frac{l}{r} = \frac{0.4}{0.1} = 4$$

Inertia force, $I_p = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$$= 180 \times \left(\frac{2\pi N}{60}\right)^2 \times 0.1 \left[\cos 60^\circ + \frac{\cos 120}{4}\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 \rightarrow (52.4)^2$$

$$= 18.53 \times 10^3 \text{ N}$$

② The Crank pin circle radius of a horizontal reciprocating engine is 300 mm. The mass of the reciprocating parts is 250 kg. When the crank has travelled 60° from IDC. The difference b/w the driving and back pressure is 0.35 N/mm². The con rod length b/w centres is 1.2 m and the cylinder bore is 0.5 m. If the engine runs at 250 rpm ~~and if the effect of piston rod~~. Calculate (1) pressure on slide bars. (2) Thrust in the con rod (3) Tangential force on the crank pin. (4) Turning Moment on the crank shaft.

$r = 300$ mm $= 0.3$ m, $m_R = 250$ kg, $\theta = 60^\circ$, $P = 0.35$ N/mm².

$l = 1.2$ m, $D = 0.5$ m $= 500$ mm. $N = 250$ rpm

$$\omega = 26.2 \text{ rad/sec} \leftarrow \left(\frac{2\pi N}{60}\right)$$

Net load on piston, $F_{piston} = P \times \frac{\pi}{4} D^2 = 68730$ N.

$$n = l/r = \frac{1.2}{0.3} = 4$$

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 250 \times (26.2)^2\, 0.3 \left[\cos 60 + \frac{\cos 120}{4}\right)$$

$$= 19306 \ N.$$

piston effort, $F_P = F_{piston} - F_I \quad = 49424 \ N$

(1) pressure on slide bar

$n \sin\phi = \sin\theta. \Rightarrow \quad \phi = 12.5^\circ$

$F_N = F_P \tan\phi \quad = 10.96 \ kN$

(2) Thrust in the con rod

$$F_Q = \frac{F_P}{\cos\phi} = \frac{49.424}{\cos 12.5^\circ} = 50.62 \ kN$$

(3) Tangential force on the crank pin

$F_T = F_Q \sin(\theta + \phi) = 48.28 \ kN$

(4) Turning Moment on the crank shaft.

$T = F_T \times r \quad = 14.484 \ kNm$

The Crank and Connecting rod of a petrol engine, running at 1800 rpm are 50 mm and 200 mm respectively. The diameter of the piston is 80 mm and the mass of the reciprocating parts is 1 kg. the pressure on the piston is 0.7 N/mm^2, ~~and~~ when the crank has turned through 33° from the IDC. Determine (i) Net ~~load~~ Force on the piston. (ii) Thrust in the connecting rod (iii) Reaction between piston and cylinder (and) (iv) The engine speed at which the above values become zero.

GD

$N = 1800$ rpm, $\omega = \frac{2\pi \times 1800}{60} = 188.52$ rad/sec

$r = 50$ mm $= 0.05$ m

$l = 200$ mm $= 0.2$ m

$D = 80$ mm, $m_R = 1$ kg, $P = 0.7$ N/mm^2, $\theta = 33°$

$$F_{piston} \Rightarrow \frac{\pi}{4} D^2 \times P = \frac{\pi}{4} \times 80^2 \times 0.7 \Rightarrow \underline{3520 \text{ N}}$$

$$n = \frac{l}{r} = \frac{200}{50} = 4$$

$$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right] = \underline{1671 \text{ N}}$$

piston effect $\Big\} \rightarrow$ $F_p = F_{piston} - F_I = \underline{1849 \text{ N}}$

$$\sin\phi = \frac{\sin\theta}{n} \Rightarrow \underline{\phi = 7.82°}$$

Thrust in Con. rod.

$$F_Q = \frac{F_P}{\cos\phi} = 1866.3 \text{ N}$$

(3) Reaction b/w the piston & cylinder

$$F_N = F_P \tan\phi = 1849 \tan 7.82^\circ$$

$$F_N = 254 \text{ N}$$

(4) Engine speed at which the above values will become zero

$$m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right] = \frac{\pi}{4} D^2 \times P.$$

$$\omega^2 = 74894$$

$$\omega = 273.6 \text{ rad/sec.}$$

$$\omega = \frac{2\pi N}{60} \Rightarrow N = 2612 \text{ rpm}$$

(3) A vertical double acting ~~cylinder~~ steam engine has a cylinder 300 mm diameter and 450 mm stroke and runs at 200 rpm. The reciprocating parts has a mass of 225 kg and the piston rod is 50 mm diameter. The connecting rod is 1.2 m long. When the crank has turned through 125° from the top dead centre, the steam pressure above the piston is 30 kN/m² and below the piston is 1.5 kN/m². Calculate the effective turning moment on the crank shaft.

$D = 300$ mm $= 0.3$ m , $r = \frac{450}{2} = 225$ mm $= 0.225$ m , $N = 200$ rpm,

$\omega = \frac{2\pi N}{60} = 20.95$ r/sec

$m_R = 225$ kg, $d = 50$ mm $= 0.05$ m.

$l = 1.2$ m , $\theta = 125°$, $P_1 = 30$ kN/m² $= 30 \times 10^3$ N/m² | $P_2 = 1.5$ kN/m² $= 1.5 \times 10^3$ N/m².

$= P_1 A_1 - P_2 A_2$

Force on the piston, $F_{piston} = P_1 \left[\frac{\pi}{4} D^2\right] - P_2 \left[\frac{\pi}{4}(D^2 - d^2)\right]$.

$= 2018$ N

$n = l/r = 5.33$

Inertia force, $F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$= -14172$ N

piston effort, $F_P = F_{Pgm} - F_I + m_R g$.

$= 2018 - (-14172) + [225 \times 9.81]$

$= 18397$ N

Turning effect on the crank shaft

$= F_T \times r$

$T = F_Q \sin(\theta + \phi)$

$\frac{F_P}{\cos\phi} \times \sin(\theta+\phi)$

$$T = \frac{F_p \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{18397 \sin(125^\circ + 8.84^\circ)}{\cos 8.84^\circ} \times 0.225$$

$\sin\phi = \frac{\sin\theta}{n}$

$\sin\phi = 0.1537$

$\phi = 8.84^\circ$

$$\boxed{T = 3021.6 \text{ Nm}}$$

(4) During a trial on steam engine, it is found that the acceleration of the piston is 36 m/sec² when the crank has moved 30° from the inner dead centre position. The net effective steam pressure on the piston is 0.5 N/mm² and the frictional resistance is equivalent to a force of 600 N. The diameter of the piston is 300 mm and the mass of the reciprocating parts is 180 kg. If the length of the crank is 300 mm and the ratio of the connecting rod length to the crank length is 4.5. Find (1) Reaction on the guide bars (2) Thrust on the crank shaft bearing (3) Turning moment on the crank shaft.

GD $a_p = 36\ m/sec^2$ $m_R = 180\ kg$

$\theta = 30^\circ$ $r = 300\ mm = 0.3\ m$

$p = 0.5\ N/mm^2$ $n = l/r = 4.5$

$R_F = 600\ N$

$D = 300\ mm$

Load on piston

$$F_{piston} = p \times \frac{\pi}{4} D^2 = 35350\ N$$

~~piston effort~~ Inertia force

$$F_I = m_R a_p = 180 \times 36 = 6480\ N$$

piston effort

$$F_p = F_{piston} - F_I - F_F$$

$$= 35350 - 6480 - 600$$

$$= 28270\ N$$

$$\sin\phi = \frac{\sin\theta}{n} \Rightarrow \phi = 6.38^\circ$$

Reaction on guide bars

$$F_N = F_p \tan\phi = 3.16\ kN$$

Trust on the crank shaft bearing

$$F_B = \frac{F_p \cos(\theta+\phi)}{\cos\phi} = 22.9\ kN$$

Turning Moment on the crank shaft

$$T = \frac{F_p \sin(\theta+\phi)}{\cos\phi} \times r = 5.06\ kNm$$

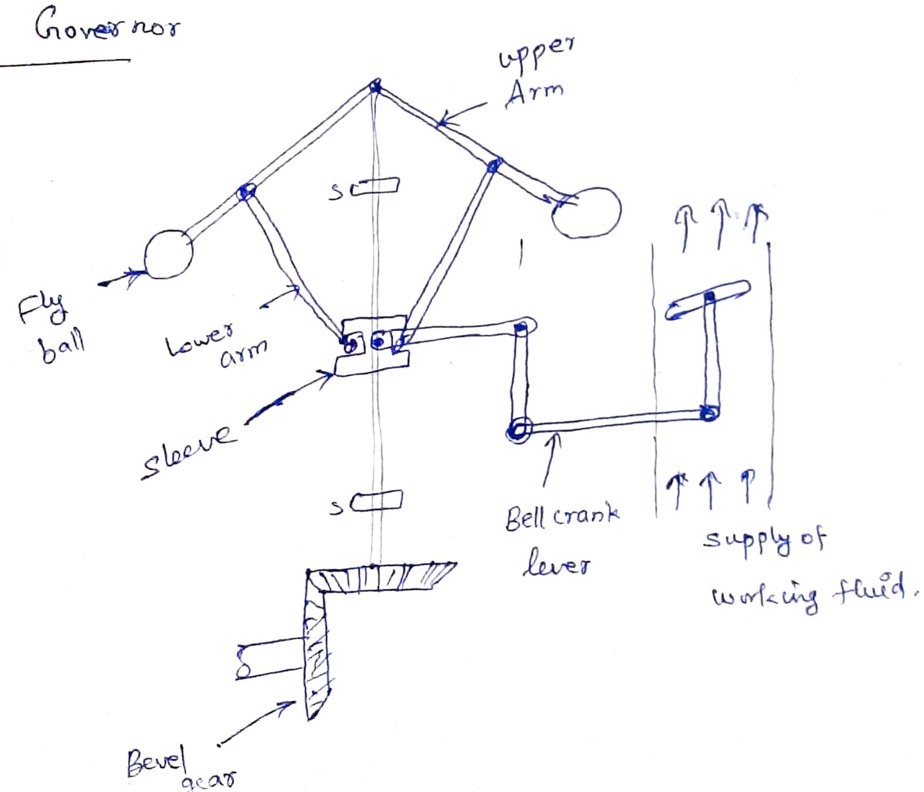
Governors

→ The function of a governor is to regulate the mean speed of an engine.

Types of governors

1. Inertia Governor
2. centrifugal Governors.
   - pendulam type (watt governor)
   - Loaded type
     - Dead weight
       - porter
       - proell
     - Spring controlled
       - Hartnell
       - Harting
       - Wilson hartnell
       - pickering.

Centrifugal Governor



Terms

Height of a governor

→ It is the vertical distance from the centre of the ball to a point where the axes of the arms intersect on the spindle axis. It is denoted by "h".

Equilibrium speed

→ It is the speed at which the governor balls, arms etc in complete equilibrium and the sleeve does not tend to move upwards (or) downwards.

Mean equillibrium speed

→ It is the speed at mean position of the balls (or) the sleeve.

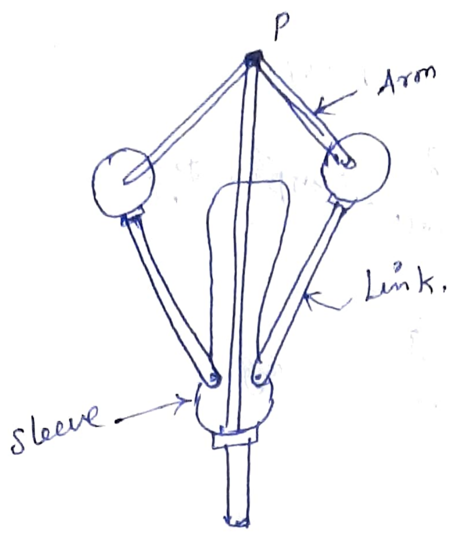
Max & Min equillibrium speeds

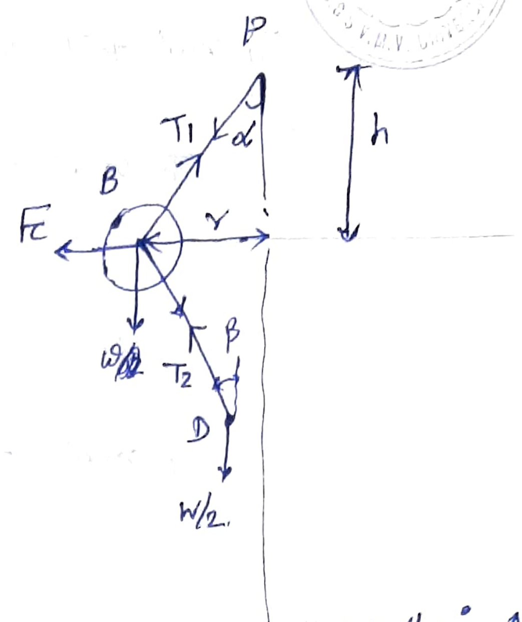
→ The speeds at which the Max and Min radius of rotation of the balls, without tending to move either way are known as Max and min equillibrium speeds respectively.

Sleeve lift

→ It is the vertical distance which the sleeve travels due to change in equillibrium speed.

porter Governor





m – Mass of each ball in kg, w – Mass of each ball in N.

M – " " sleeve in kg, W – Mass of sleeve in N

r → Radius of rotation in "m"

h → height of governor in "m"

N → Speed of the balls in rpm.

ω → Angular speed of the ball → $\frac{2\pi N}{60}$ rad/sec.

F_c → centrifugal force acting on the ball $= m\omega^2 r$ in "N"

T_1 → Force in the arm in N

T_2 → Force in the link in N.

α → Angle of inclination of the arm to the verticale

β → " " " " link " "

Consider D is in equilibrium

$$W/2 = T_2 \cos\beta.$$

$$T_2 = \frac{W}{2\cos\beta} = \frac{Mg}{2\cos\beta}$$

point B is in equilibrium

$\underline{\underline{\Sigma V = 0}}$

$T_1 \cos\alpha - T_2 \cos\beta - W = 0$

$T_1 \cos\alpha = T_2 \cos\beta + W$

$= \frac{Mg}{2\cancel{\cos\beta}} \times \cancel{\cos\beta} + mg$

$\therefore T_1 \cos\alpha = \underline{\underline{\frac{Mg}{2}}} + mg$

$\underline{\underline{\Sigma H = 0}}$

$T_1 \sin\alpha + T_2 \sin\beta = F_c$

$T_1 \sin\alpha + \frac{Mg}{2\cos\beta} \times \sin\beta = F_c$

$T_1 \sin\alpha + \frac{Mg}{2} \tan\beta = F_c$

$T_1 \sin\alpha = F_c - \frac{Mg}{2} \tan\beta$

$\div T_1 \cos\alpha$

$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_c - \frac{Mg}{2}\tan\beta}{T_1 \cos\alpha}$

$\tan\alpha = \frac{F_c - \frac{Mg}{2}\tan\beta}{\frac{Mg}{2} + mg}$

$\left(\frac{Mg}{2} + mg\right) \tan\alpha = F_c - \frac{Mg}{2}\tan\beta.$

$\frac{Mg}{2} + mg = \frac{F_c}{\tan\alpha} - \frac{Mg}{2} \frac{\tan\beta}{\tan\alpha}.$

Assume $\boxed{\frac{\tan\beta}{\tan\alpha} = q}$ $\boxed{\tan\alpha = \frac{r}{h}}$

$$\frac{Mg}{2} + mg = \left(m\omega^2 r \times \frac{h}{r}\right) - \left(\frac{Mg}{2} \times q\right)$$

$$m\omega^2 h = \frac{Mg}{2} + mg + \frac{Mg}{2} q.$$

$$m\omega^2 h = mg + \frac{Mg}{2}(1+q).$$

$$\omega^2 h = \frac{mg + \frac{Mg}{2}(1+q)}{mh}$$

$$\left[\frac{2\pi N}{60}\right]^2 = \frac{g\left[m + \frac{M}{2}(1+q)\right]}{mh} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{9.81 \times 60^2}{h \times 4 \times \pi^2}$$

$$\boxed{N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}}$$

Note (i) The length of the arms are equal to the length of links and the points P & D lies on the same vertical line,

$\tan\alpha = \tan\beta$, $\boxed{q = 1}$

$$\therefore N^2 = \frac{m + M}{m} \times \frac{895}{h}$$

(ii) Friction force acting on the sleeve is "N"

$$N^2 = \frac{mg + \left[\frac{Mg \pm F}{2}\right](1+q)}{mg} \times \frac{895}{h} \quad \text{when } (q=1)$$

$$N^2 = \frac{mg + [Mg \pm F]}{mg} \times \frac{895}{h}$$

(+) sleeve moves upward

(−) sleeve moves downward

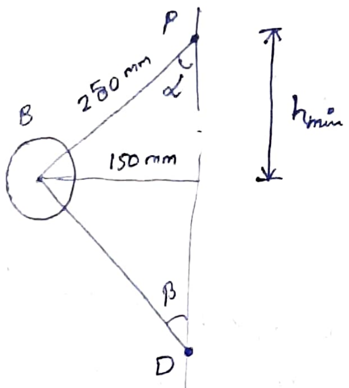
① A porter Governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at Max speed. Find the Min and Max speeds and range of speed of the governor.

arm length = 250 mm. | r_{min} = 150 mm

m = 5 kg. | r_{max} = 200 mm

M = 25 kg.

Min speed

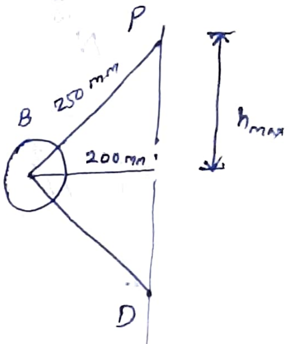


$\alpha = \beta$, $q = 1$

$h_{min} = \sqrt{250^2 - 150^2} = 200$ mm

$h = 0.2$ m

Max speed



$\alpha = \beta$, $q = 1$

$h_{max} = \sqrt{250^2 - 200^2} = 150$ mm

$h = 0.15$ m

$$N_{min}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{min}} = \cancel{163.85}\ 26850$$

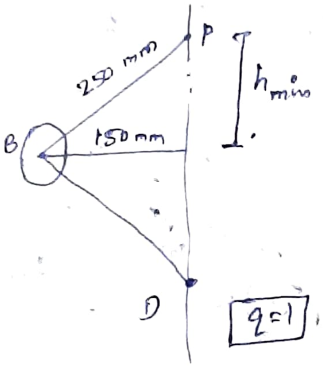
$$\boxed{N_{min} = 163.85 \text{ rpm}}$$

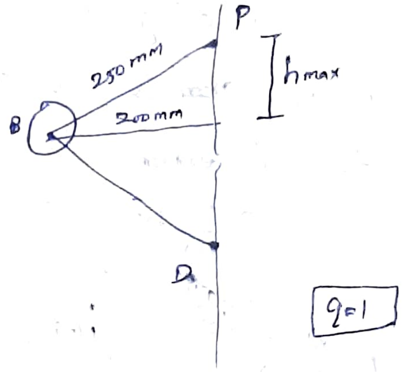
$$N_{max}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{max}} \Rightarrow \boxed{N_{max} = 189.2 \text{ rpm}}$$

Range of speed $= N_{max} - N_{min} = 25.35$ rpm

② The arms of a porter Governor are each 250 mm long and pivoted on the governor axis. The Mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the ball is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for max speed. Deter the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

(i) ~~with out~~ No friction force





$h_{max} = 0.15\ m.$

$h_{min} = 0.2\,m$

$$N_{min}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{min}} = 31325 \Rightarrow \boxed{N_{min} = 177\ rpm}$$

$$N_{max}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{max}} = 41767 \Rightarrow \boxed{N_{max} = 204.4\ rpm}$$

$$\text{Range of speed} = N_{max} - N_{min} = 27.4\ rpm$$

(ii) Friction force, $F = 20\,N = 2\ kg$

$$N_{min}^2 = \frac{m + \left[\frac{M-F}{2}\right](1+q)}{m} \times \frac{895}{h_{min}} \Rightarrow \boxed{N_{min} = 172\ rpm}$$

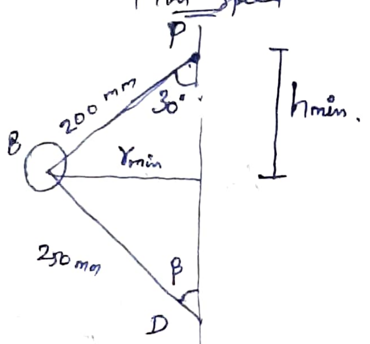
$$N_{max}^2 = \frac{m + \left(\frac{M+F}{2}\right)(1+q)}{m} \times \frac{895}{h_{max}}$$

$$\boxed{N_{max} = 210 \text{ rpm}}$$

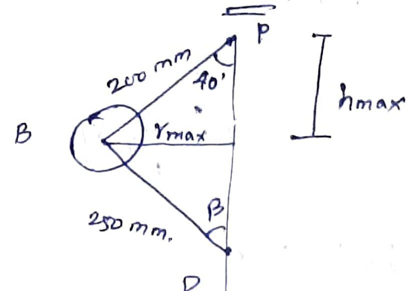
Speed Range = 38 rpm

(3) In an engine Governor of the porter type, the upper and lower arms are 200 mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and the frictional force is 24 N at the sleeve. If the limiting inclinations of the upper arms to the vertical ~~arm~~ is 30° and 40°. Find Range of speed of the Governor.

Min speed



Max speed



$r_{min} = \sin 30 \times 200 = 100$ mm

$h_{min} = \cos 30 \times 200 = 0.173$ mm $= 0.173$ m

$\sin \beta = \frac{r_{min}}{250} \Rightarrow \beta = 23.57$

$q = \frac{\tan\beta}{\tan\alpha} = 0.756$

$$N_{min}^2 = \frac{m + \left(\frac{M-F}{2}\right)(1+q)}{m} \times \frac{895}{h_{min}}$$

$N_{min} = 183.3$ rpm

$r_{max} = \sin 40 \times 200 = 126$ mm

$h_{max} = \cos 40 \times 200 = 0.153$ m

$\sin \beta = \frac{r_{max}}{250} \Rightarrow \beta = 30.79$

$q = \frac{\tan\beta}{\tan\alpha} = 0.71$

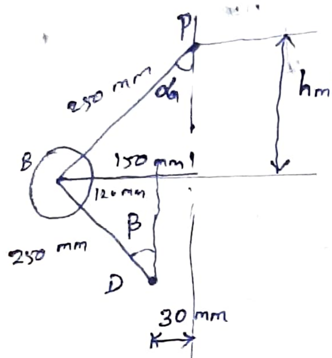
$$N_{max}^2 = \frac{m + \left(\frac{M+F}{2}\right)(1+q)}{m} \times \frac{895}{h_{max}}$$

$N_{max} = 222$ rpm

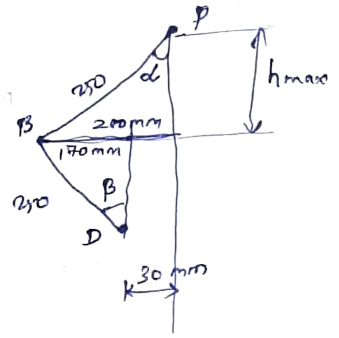
Range of speed = 38.7 rpm

(4) A porter Governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg, and the sleeve has a mass of 50 kg. The extreme radius of rotations are 150 mm and 200 mm, Determine the range of speed of the Governor,

Min speed



Max speed



$h_{max} = \sqrt{250^2 - 200^2} = 0.15\ m$

$h_{min} = \sqrt{250^2 - 150^2} = 0.2\ m$

$\tan\alpha = \frac{200}{150} = 1.333$,

$\tan\alpha = \frac{150}{200} = 0.75$

$\sin\beta = \frac{170}{250} \Rightarrow \beta =$

$\sin\beta = \frac{120}{250} =$ $\beta =$

$q = \frac{\tan\beta}{\tan\alpha} = 0.7$

$q = \frac{\tan\beta}{\tan\alpha} = 0.731$

$N_{max}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{max}}$

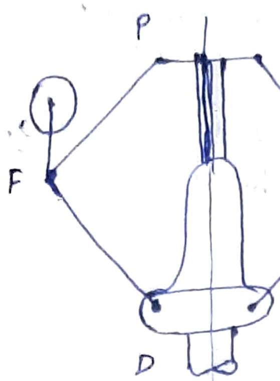
$N_{min}^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{min}}$

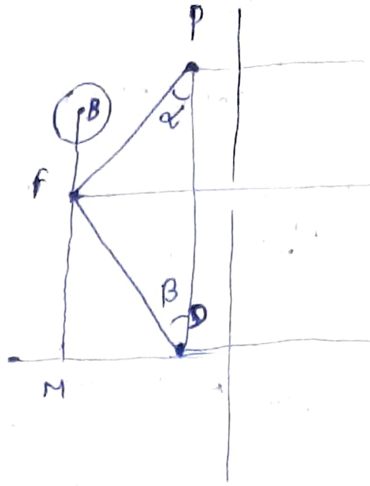
$\boxed{N_{max} = 238\ rpm}$

$\boxed{N_{min} = 208\ rpm}$

Range of speed = 30 rpm

proell Governor

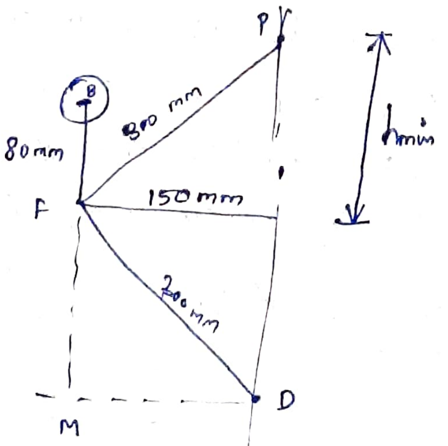




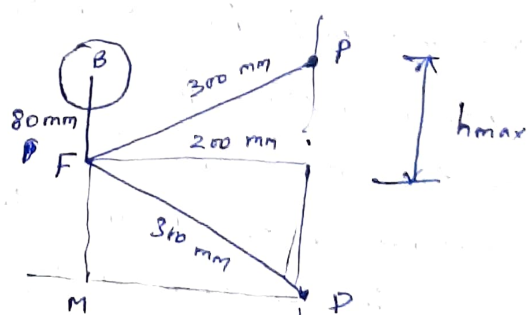
$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right] \times \frac{895}{h}$$

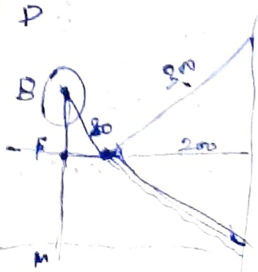
① A proell Governor has equal arms of length 300 mm The upper and lower ends of the arms are pivoted on the axis of the Governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radious of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Min speed



Max speed





Min speed

$$h_{min} = \sqrt{300^2 - 150^2} = 0.26 \text{ m}$$

$$BM = BF + FM = 0.26 + 0.08 = 0.34 \text{ m}$$

$$N_{min}^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{min}}\right] \quad (q=1)$$

$$\boxed{N_{min} = 170 \text{ rpm}}$$

Max speed

$$h_{max} = \sqrt{300^2 - 200^2} = 0.224 \text{ m}$$

$$BM = 0.08 + 0.224 = 0.304 \text{ m}$$

$$N_{max}^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_{max}}\right]$$

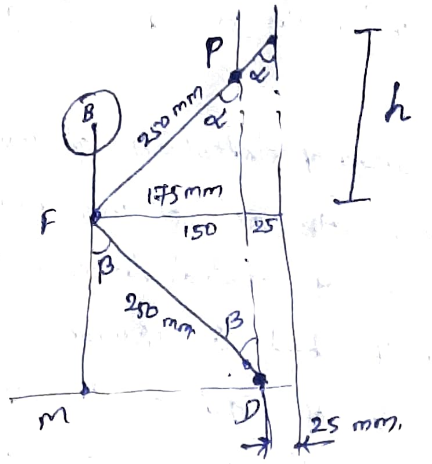
$$\boxed{N_{max} = 180 \text{ rpm}}$$

Range of speed = 10 rpm

(2) A Governor of the proell type has each arm 250 mm long. The pivots of the upper and lower arms are 25 mm from the axis. The central load acting on the sleeve has a mass of 25 kg and the each rotating ball has a mass of 3.2 kg, when the Governor sleeve is in mid position, the extension link of the lower arm is vertical and the radious of the rotation of the masses is 175 mm. If the governor speed is 160 rpm when in mid position. find

(1) The length of the extension link.

(2) Tension in the upper arm.



$$\alpha = \sin^{-1}\left[\frac{150}{250}\right] = \underline{\underline{36.87^\circ}}$$

$$\cos\alpha = \frac{h}{250} \rightarrow h = \underline{\underline{233.3}} \text{ mm} = \underline{\underline{0.233}} \text{ m}.$$

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}\right]$$

$q = 1$ $(\tan\alpha = \tan\beta)$

$$FM = \sqrt{250^2 - 150^2} = 200 \text{ mm.} = 0.2 \text{ m.}$$

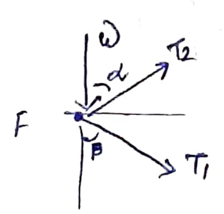
$$\boxed{BM = \cancel{0.308\ m}} \quad 0.26 \text{ m.}$$

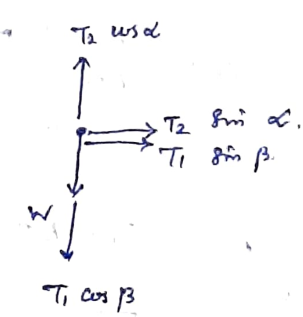
$$BM = BF + FM$$

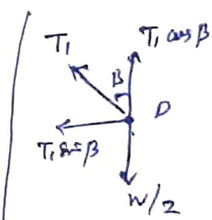
$$0.268 = BF + 0.2$$

$$\boxed{BF = \cancel{0.108}\ m} \quad 0.06 \text{ m.} = 60 \text{ mm.}$$

Tension







$\Sigma V = 0$

$$T_1 \cos\beta = \frac{W}{2}$$

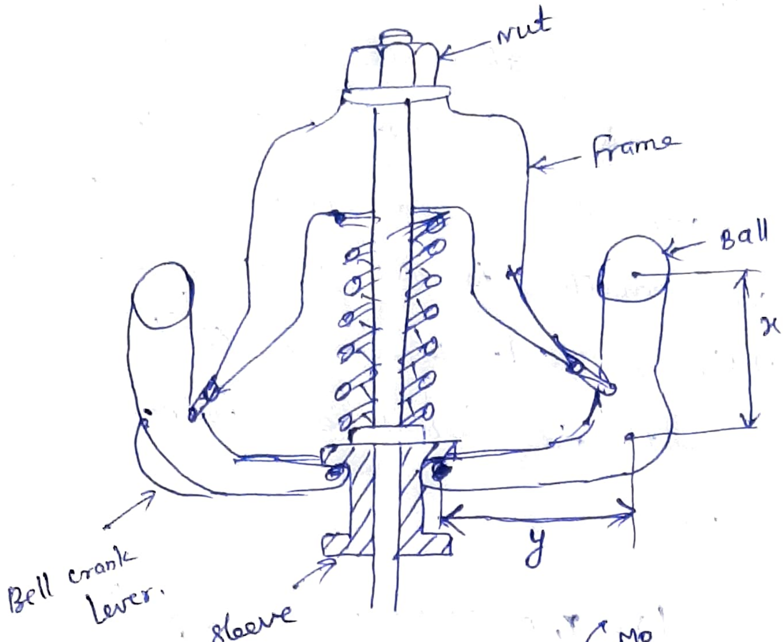
$\Sigma V = 0$

$$T_2 \cos\alpha = W + T_1 \cos\beta.$$

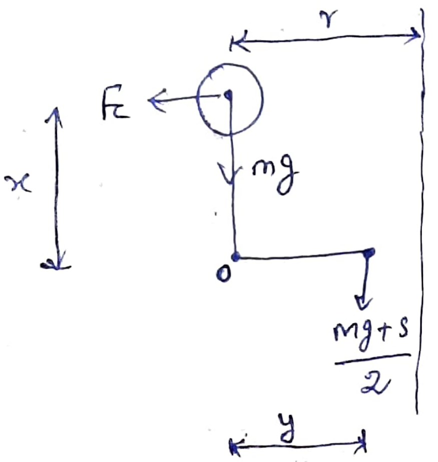
$$T_2 \cos\alpha = W + \frac{W}{2}$$

$$\boxed{T_2 = 192.5\ N}$$

Hartnell Governor



"CMo"



$$Mg + S_1 = 2F_{c_1} \times \frac{x}{y}.$$

$$Mg + S_2 = 2F_{c_2} \times \frac{x}{y}.$$

def., $h = (r_2 - r_1)\frac{y}{x}.$

$$\text{Stiffness} = \frac{S_2 - S_1}{h} = 2\left[\frac{F_{c_2} - F_{c_1}}{r_2 - r_1}\right]\left(\frac{x}{y}\right)^2$$

① A hartnell governor having a central sleeve spring and two right angled bell crank lever moves b/w 290 rpm and 310 rpm for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equillibrium speed. Determine (i) load on the spring at the lowest and highest equillibrium speeds and (ii) spring stiffness.

$N_1 = 290$ rpm

$N_2 = 310$ rpm

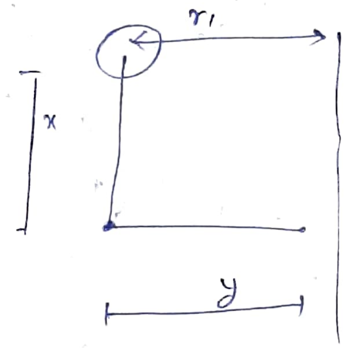
$h = 15$ mm

$y = 80$ mm

$x = 120$ mm.

$r = r_1 = 120$ mm.

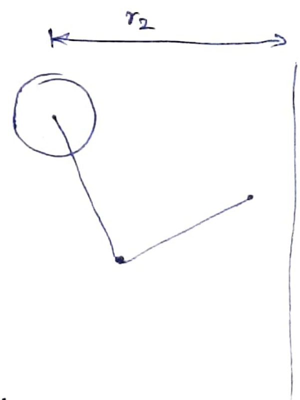
$m = 2.5$ kg.



$$\cancel{Mg}^{0} + S_1 = 2F_1 \times \frac{x}{y}.$$

$$S_1 = 2\left[2.5\left(\frac{2\pi \times 290}{60}\right)^2 \times 0.12\right] \times \frac{0.12}{0.08}$$

$$\boxed{S_1 = 831\text{ N}}$$



$$h = (r_2 - r_1)\frac{y}{x}.$$

$$r_2 = 0.142\text{ m}.$$

$$\cancel{Mg}^{0} + S_2 = 2F_2 \times \frac{x}{y}.$$

$$S_2 = 2\left[2.5\left(\frac{2\pi \times 310}{60}\right)^2 \times 0.142\right] \times \frac{0.12}{0.08}$$

$$\boxed{S_2 = 1128\text{ N}}$$

Stiffness, $s = \frac{S_2 - S_2}{h} = 19.8$ N/mm

(2) In a spring loaded Hartnell type Governor, the extreme radius of rotation of the balls are 80 mm and 120 mm. The ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 rpm. Find (1) Spring constant (or) spring stiffness (2) Initial compression of the central spring.

$r_1 = 80\ mm$ $\quad x = y,$ $\quad m = 2\ kg,$ $\quad n_1 = 400\ rpm$

$r_2 = 120\ mm$ $\quad n_2 = 420\ rpm.$

Min speed

$$Mg + S_1 = 2F_{c1} \times \frac{x}{y}$$

$$S_1 = 2F_{c1} = \underline{562\ N}$$

Max speed

$$Mg + S_2 = 2F_{c2} \times \frac{x}{y}$$

$$S_2 = 2F_{c2} = \underline{\underline{930\ N}}$$

Spring stiffness, $s = \dfrac{S_2 - S_1}{h}$ $\qquad h = (r_2 - r_1)\dfrac{y}{x}$

$h = 40\ mm$

$$\boxed{s = 9.2\ N/mm}$$

Initial Compression, $s = \dfrac{S_1}{h_1}$

$$h_1 = S_1 / s.$$

$$\boxed{h_1 = 61\ mm}$$

Sensitive Governor and sensitiveness of Governor

Consider two governors A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

The sensitiveness is defined as the ratio of the difference b/w the maximum and Minimum equilibrium speeds to the mean equilibrium speed.

$$\text{sensitiveness} = \frac{(N_2 - N_1)}{\frac{N_1 + N_2}{2}}$$

$$\boxed{\text{Sensitiveness} = \frac{2(N_2 - N_1)}{N_1 + N_2}}$$

Stability of Governor

A Governor is said to be stable when for every speed within the working range there is only one radius of rotation of the governor balls at which the governor is in equilibrium.

For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Isochronous governors

A governor is said to be isochronous when the equillibrium speed is constant for all radius of rotation of the balls within the working Range.

Range of speeed should be zero.

Hunting of Governors

A governor is said to be hunt of the speed of the engine fluctuates continously above and below the mean speed.

Torsional vibrations

Natural frequency, $f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

$\frac{T}{J} = \frac{C\theta}{l}$

$\frac{T}{\theta} = \frac{CJ}{l}$

$$\boxed{q = \frac{CJ}{l}}$$

$C \rightarrow$ Modulus of Rigidity N/m^2

$J \rightarrow$ polar moment of Inertia, $\frac{\pi}{32} d^4$ (m^4)

$q \rightarrow$ Torsional stiffness, Nm.

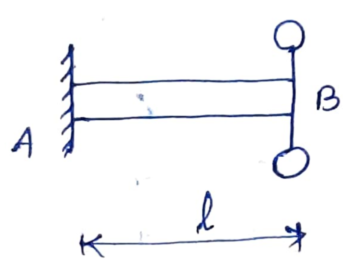
$$\boxed{I = mk^2}$$

$m \rightarrow$ mass of disc in kg.

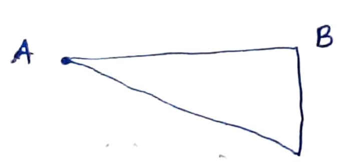
$k \rightarrow$ Radius of gyration in m.

$I \rightarrow$ Mass moment of Inertia, $kg\,m^2$.

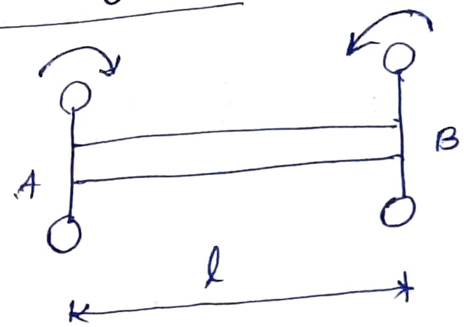
(1) Torsional vibration of a single rotor system.

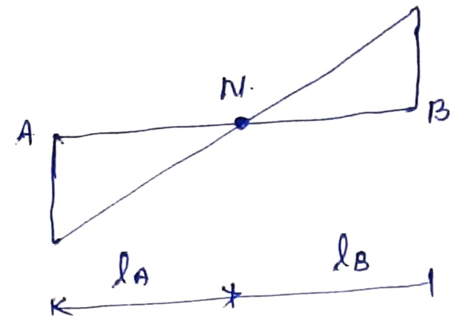


$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{lI}}.$$



② Two Rotor system





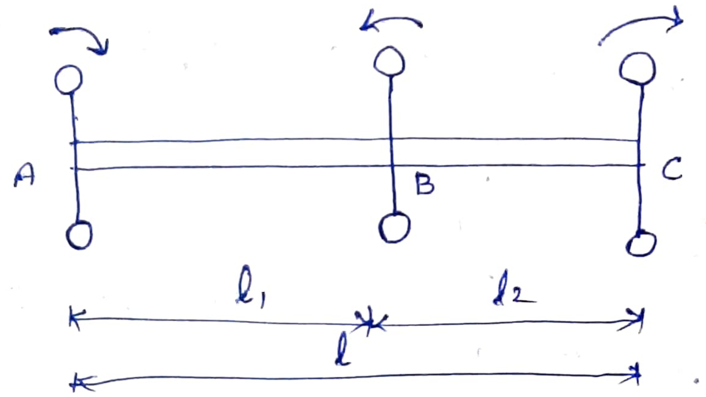
$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}} \quad , \quad f_{nB} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_B I_B}}$$

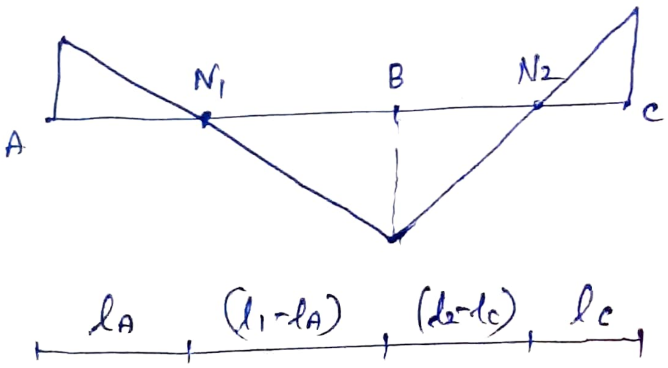
Here $f_{nA} = f_{nB}$

$$\therefore \boxed{l_A I_A = l_B I_B}$$

$$\boxed{l = l_A + l_B}$$

③ Three rotor system





$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}} \quad , \quad f_{nC} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_c I_c}}$$

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_c}\right]}$$

Here, $f_{nA} = f_{nB} = f_{nC}$

$$\boxed{l = l_1 + l_2}$$

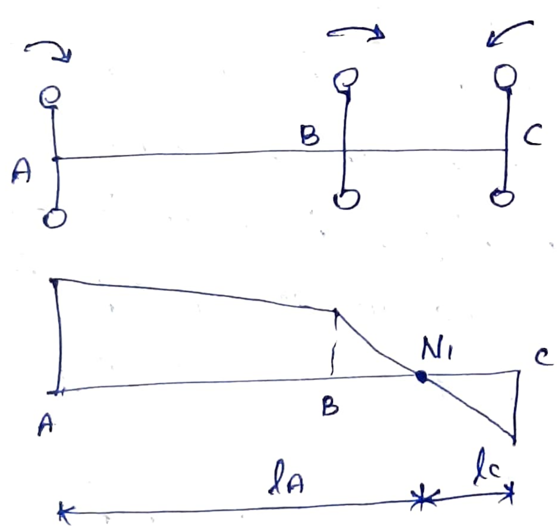
$$\boxed{l_A I_A = l_c I_c}$$

$$\frac{1}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_c}\right] = \frac{1}{l_c I_c} = \frac{1}{l_A I_A}.$$

single note system

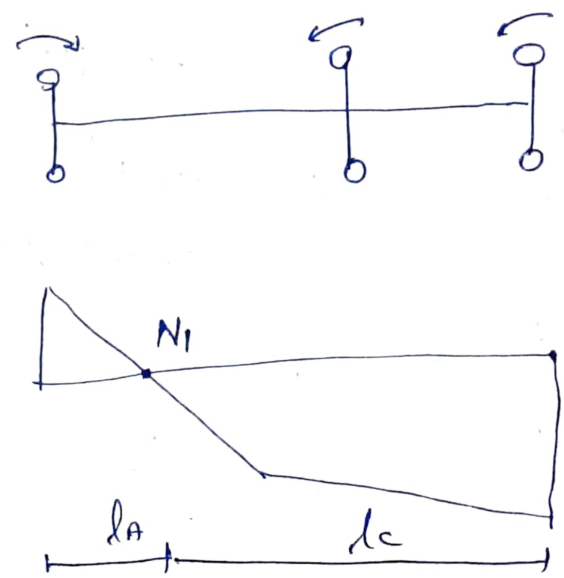
Note

1.



$[l_A > l_1]$

2.



$[l_c > l_2]$

(1) A shaft of 100 mm diameter and 1m long has one of its end fixed and the other end carries a disc of mass 500 kg at a radius of gyration of 450 mm. The modulus of Rigidity for the shaft material is 80 GN/m². Determine the frequency of torsional vibration.

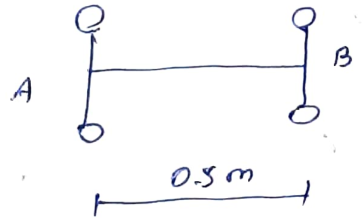
$J = \frac{\pi}{32} d^4 = 9.82 \times 10^{-6}\ m^4$.

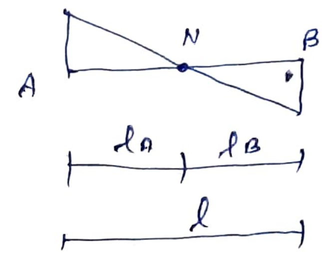
$I = mk^2 = 500\ (0.45)^2 = 101.25\ kg.m^2$.

$f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{lI}} = \sqrt{\frac{80\times10^9 \times 9.82\times10^{-6}}{1\times 101.25}}$

$\boxed{f_n = 14\ Hz}$

(2) The Two rotors A & B are attached to the end of a shaft 500 mm long. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of the rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in diameter and the modulus of rigidity is 80 GN/m². Find the position of the node and frequency of torsional vibration.





$I_A = m_A k_A^2$

$= 27\ kg\ m^2$

$I_B = m_B k_B^2$

$= 101.25\ kg\ m^2$

$C = 80\times10^9\ N/m^2$

$J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 0.07^4$

$= 2.35\times10^{-6}\ m^4$

$$l_A = \frac{I_B}{I_A} l_B$$

$$\boxed{l_A = 3.75\, l_B}$$

$$l = l_A + l_B$$

$$0.5 = 3.75\, l_B + l_B$$

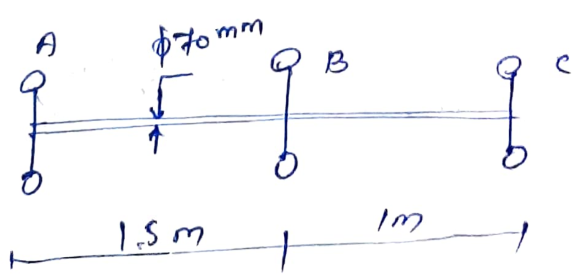
$$\boxed{l_B = 0.1\ m} \quad \boxed{l_A = 0.4\ m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{C\,J}{l_A\, I_A}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{80\times10^9 \times 2.38\times10^{-6}}{0.4\times 27}}$$

$$\boxed{f_n = 21\ Hz}$$

③ A single cylinder oil engine drives directly a centrifugal pump. The rotating mass of the engine, flywheel and the pump with the shaft is equivalent to a three rotor system as shown in figure. The mass moment of Inertia of the rotor A, B & C are 0.15, 0.3 and 0.09 kgm^2. Find the natural frequency of the torsional vibration. Take $C = 84\ KN/mm^2$
$= 84\times10^9\ N/m^2$



$$\frac{1}{l_A I_A} = \frac{1}{l_c I_c} = \frac{1}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_c}\right]$$

$$l_A I_A = l_c I_c$$

$$l_A = \frac{0.09}{0.15} l_c = \underline{0.6 l_c}$$

$$\frac{1}{l_c I_c} = \frac{1}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_c}\right] \qquad \left[\begin{matrix} l_1 = 1.5 \\ l_2 = 1 \end{matrix}\right]$$

$$\frac{1}{0.09 l_c} = \frac{1}{0.3}\left[\frac{1}{1.5 - l_A} + \frac{1}{1 - l_c}\right]$$

$$\frac{0.3}{0.09 l_c} = \frac{1}{1.5 - 0.6 l_c} + \frac{1}{1 - l_c}$$

$$\frac{10}{3 l_c} = \frac{1 - l_c + 1.5 - 0.6 l_c}{(1.5 - 0.6 l_c)(1 - l_c)}$$

$$\frac{10}{3 l_c} = \frac{2.5 - 1.6 l_c}{1.5 - 0.6 l_c - 1.5 l_c + 0.6 l_c^2}$$

$$10.8 l_c^2 - 28.5 l_c + 15 = 0$$

(i) $l_c = 1.91$ m & $l_c = 0.726$ m. (ii)

$l_A = 1.146$ m & $l_A = 0.4356$ m.

① $l_A < l_1$ & $l_c > l_2$ — Single node system.

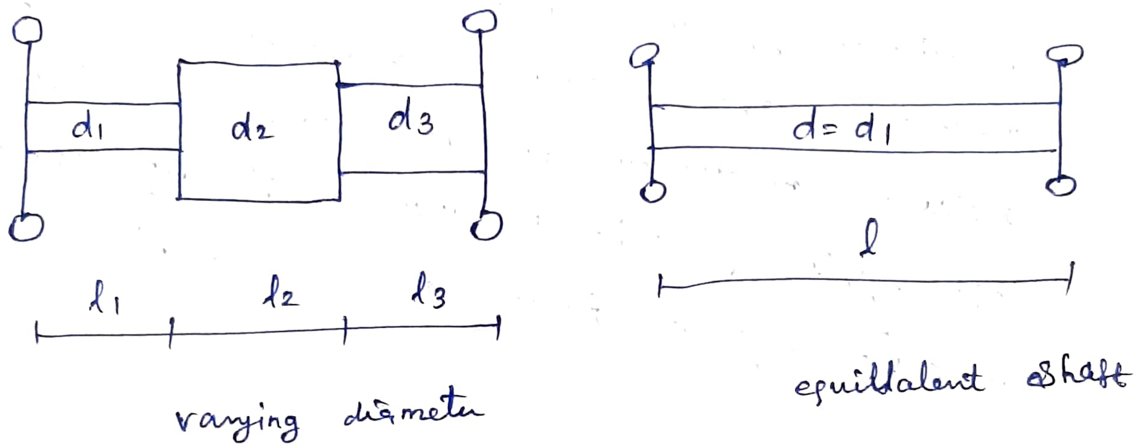
② $l_A < l_1$ & $l_c < l_2$ — Two node system.

$$J = \frac{\pi}{32} d^4 = 2.36 \times 10^{-6}\ m^4$$

single node system, $f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{l_A I_A}} = 171\ Hz$

Two node system, $f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{l_A I_A}} = 277\ Hz$

Torsionally equivalent shaft



Total angle of twist, $\theta = \theta_1 + \theta_2 + \theta_3$

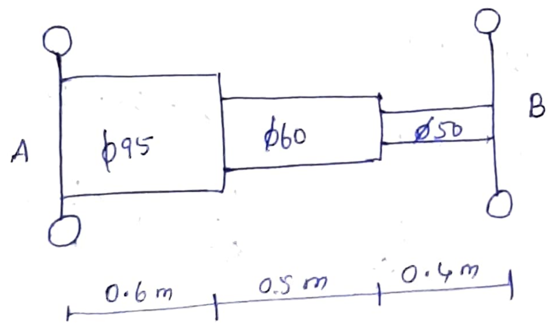
$$\frac{\cancel{T} l}{\cancel{C} J} = \frac{\cancel{T_1} l_1}{\cancel{C_1} J_1} + \frac{\cancel{T_2} l_2}{\cancel{C_2} J_2} + \frac{\cancel{T_3} l_3}{\cancel{C_3} J_3}$$

$$\frac{l}{\cancel{\frac{\pi}{32}} d^4} = \frac{l_1}{\cancel{\frac{\pi}{32}} d_1^4} + \frac{l_2}{\cancel{\frac{\pi}{32}} d_2^4} + \frac{l_3}{\cancel{\frac{\pi}{32}} d_3^4}$$

$$\boxed{l = l_1 + l_2 \left[\frac{d_1}{d_2}\right]^4 + l_3 \left[\frac{d_1}{d_3}\right]^4}$$

① The steel shaft 1.5 m long is 95 mm in diameter for the first 0.6 m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4 m of its length. The shaft carries two flywheels at two ends, the first having a mass of 900 kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having a mass of 700 kg and 0.55 m radius of gyration located at the other end. Determine the location of the node and the natural frequency of free torsional vibration of the system. The modulus of rigidity of shaft material may be taken as 80 GN/m².

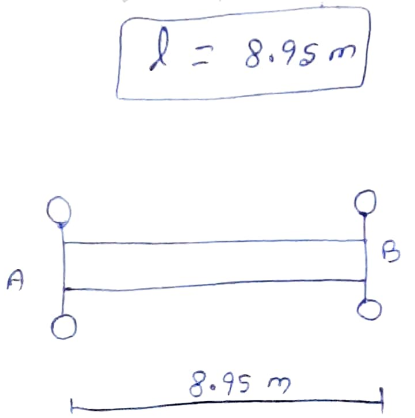
29 - 26 same
AS - 10, 14
35, 36
pres - 60



$(d = d_1)$

$$l = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4 + l_3 \left(\frac{d_1}{d_3}\right)^4$$

$\boxed{l = 8.95\,m}$ $\boxed{d = 95\,mm}$



$I_A = m_A k_A^2 = 650 \; kgm^2$

$I_B = m_B k_B^2 = 212 \; kgm^2$

~~$I_C = m_C k_C^2 = 8$~~

$J = \frac{\pi}{32} \times 0.095^4 = 8 \times 10^{-6} \; m^4$

$$\frac{1}{l_A I_A} = \frac{1}{l_B I_B}$$

$$l = l_A + l_B$$

$$l_A I_A = l_B I_B$$

$$8.95 = 0.326\, l_B + l_B$$

$$\boxed{l_A = 0.326\, l_B}$$

$$\boxed{\begin{aligned} l_B &= 6.75\ m \\ l_A &= 2.2\ m \end{aligned}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}} = \frac{1}{2\pi}\sqrt{\frac{80\times10^{9}\times 8\times10^{-6}}{2.2\times 650}}$$

$$\boxed{f_n = 3.37\ Hz}$$

position of node for original shaft

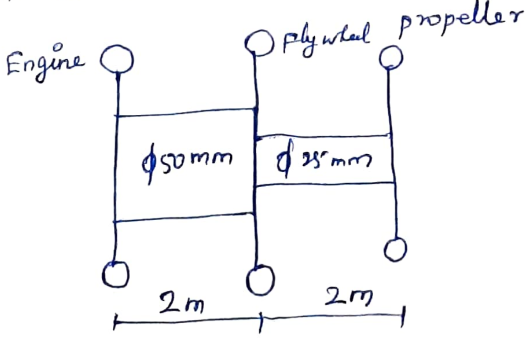
$$l_A = l_1 + (2.2 - l_1)\left(\frac{d_2}{d_1}\right)^4$$

$$\boxed{l_A = 0.855\ m.}$$

$$l_B = 1.5 - 0.855$$

$$\boxed{l_B = 0.645\ m}$$

(6) A 4 cylinder engine and flywheel coupled to a propeller are a 3 rotor system as shown in fig. in which the engine is equivalent to a rotor moment of Inertia 800 kg m^2. The fly wheel to a second rotor of 320 kgm^2 and the propeller to a third rotor of 20 kg m^2. The modulus of rigidity is 80 GN/m^2. and determine natural frequency of torsional vibration and position of the nodes.



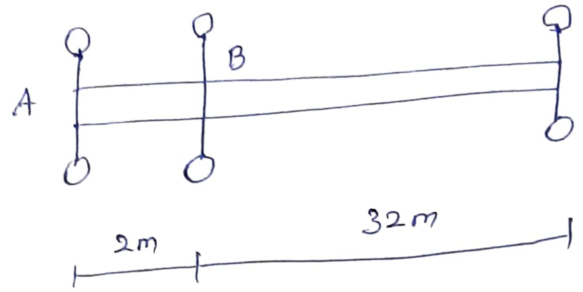
equivalent length

$$l = l_1 + l_2 \left(\frac{d_1}{d_2}\right)^4$$

$$= 2 + 2 \left(\frac{0.05}{0.025}\right)^4$$

$$\boxed{d = 0.05\ m}$$

$$= \underline{34\ m.}$$



$I_A = 800\ kgm^2$

$I_B = 320\ kgm^2$

$I_C = 20\ kgm^2$

$$l_A I_A = l_c I_c$$

$$\boxed{l_A = 0.025\, l_c}$$

$$\frac{1}{l_c I_c} = \frac{1}{I_B}\left[\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_c}\right]$$

(i) $l_c = 34.42\ m$ & $l_A = 0.86\ m$

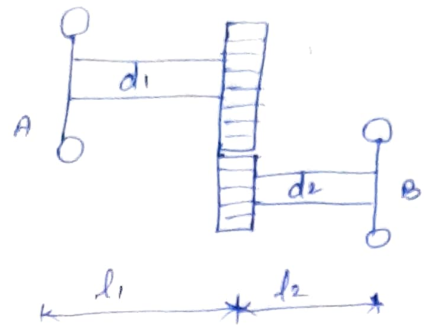
(ii) $l_c = 20.88\ m$ & $l_A = 0.52\ m$

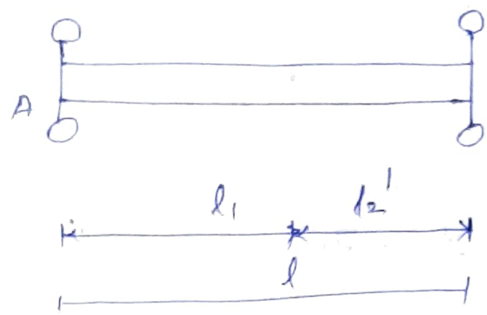
(i) $\left.\begin{array}{l} l_A < l_1 \\ l_c > l_2 \end{array}\right\} \rightarrow$ single node system. , $f_n = \underline{1.345\ Hz.}$

(ii) $\left.\begin{array}{l} l_A < l_1 \\ l_c < l_2 \end{array}\right\} \rightarrow$ Two node system. $f_n = \underline{1.73\ Hz}$

<u>position of Nodes.</u> (ii) $l_A = 0.52\ m$, $l_c = l_c\left(\frac{d_2}{d_1}\right)^4 = \underline{1.3\ m}$

Free Torsional vibrations of a Geared system



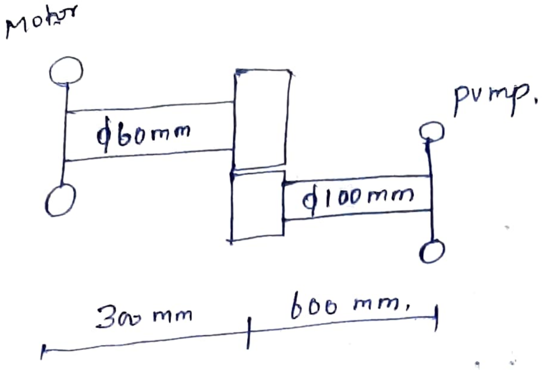


$$l = l_1 + G^2 l_2 \left(\frac{d_1}{d_2}\right)^4$$

$$I_B' = I_B / G^2$$

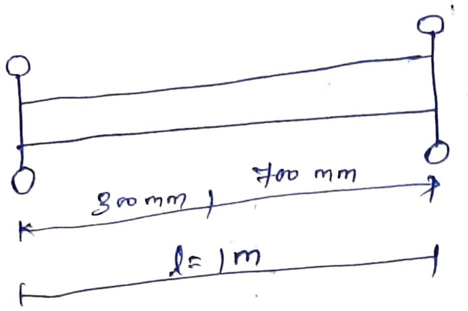
$$G = \frac{N_A}{N_B} = \frac{\omega_A}{\omega_B}$$

① A motor drives a centrifugal pump through gearing, the pump speed being one third that of the motor. The shaft from the motor to the pinion is 60 mm diameter and 300 mm long. The moment of Inertia of the motor is 400 kg.m². The impeller shaft is 100 mm diameter and 600 mm long. The moment of Inertia of the impeller is 1500 kg m². Determine the frequency of the torsional vibration of the system. Take C = 80 GN/m².



$I_A = 400\ kg\,m^2$

$I_B = 1500\ kg\,m^2$



$$l = l_1 + l_2\, G^2 \left(\frac{d_1}{d_2}\right)^4 = 1000\ mm.$$

$$I_B' = 1500/G^2$$

$$= 166.7\ kgm^2$$

$$l_A I_A = l_B I_B'$$

$$l = l_A + l_B = 1$$

$$l_A \times 400 = (1 - l_A) \times 166.7$$

$$\boxed{\begin{aligned} l_A &= 0.294\ m \\ l_B &= 0.706\ m \end{aligned}}$$

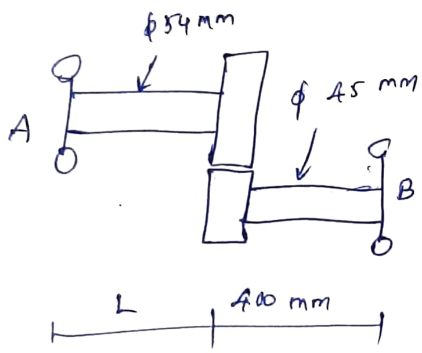
$$J = \frac{\pi}{32} \times 0.06^4$$

$$= 1.27 \times 10^{-6}\ m^4$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}} = 4.7\ Hz$$

(2) An electric motor is to drive a centrifuge, running at four times the motor speed through a spur gear and pinion. The steel shaft from the motor to the gear wheel is 54 mm diameter and "L" meter long, the shaft from the pinion to the centrifuge is 45 mm diameter and 400 mm long. The masses and radius of gyration of motor and centrifuge are respectively 37.5 kg, 100 mm; 30 kg and 140 mm.

Find the value of "L" if the gears are to be at the node for torsional vibration of the system and hence determine the frequency of torsional vibration. $C = 84\ GN/m^2$.



$N_B = 4 N_A$

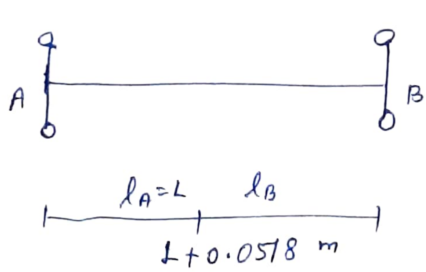
$\frac{N_A}{N_B} = \frac{1}{4} = 0.25$

$\boxed{G = 0.25}$

$I_A = m_A k_A^2$

$I_B = m_B k_B^2$

$l = L + l_2 \left(\frac{d_1}{d_2}\right)^4 G^2 = L + 0.0518\ m$



$I_B' = I_B / G^2$

$= 9.4\ kg\ m^2$

$l = l_A + l_B$

$\boxed{l_B = 0.0518\ m}$

$l_A I_A = l_B I_B'$

$\boxed{L = l_A = 1.3\ m}$

$f_n = \frac{1}{2\pi} \sqrt{\frac{CJ}{l_A I_A}} = 60.4\ Hz$

VIBRATIONS

→ when elastic bodies such as spring, a beam and a Shaft are displaced from the equillibrium position by the application of external forces, and then released, they execute a vibratory motion.

Terms

period of vibration
(or)
Time period → It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds.

cycle → It is the motion completed during one time periode.

Frequency → It is the number of cycles described in one second. The frequency is expressed in Hertz (Hz). which is eqral to one cycle per second.

Types of vibratory motion

1. Free (or) Natural vibration:-

When no external force act on the body, after giving it an Initial displacement, then the body is said to be under free (or) natural vibrations.

The frequency of the free vibrations is called free (or) Natural frequency.

2. Forced vibrations:-

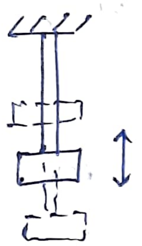
When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.

3. Damped vibrations:-

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

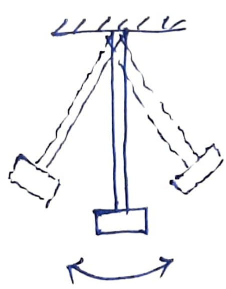
Types of free vibrations

1. Longitudinal vibrations:-



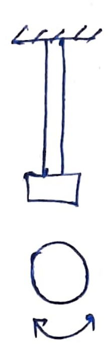
When the particles of the shaft (or) disc moves parallel to the axis of the shaft, is known as longitudinal vibrations.

2. Transverse vibrations:-



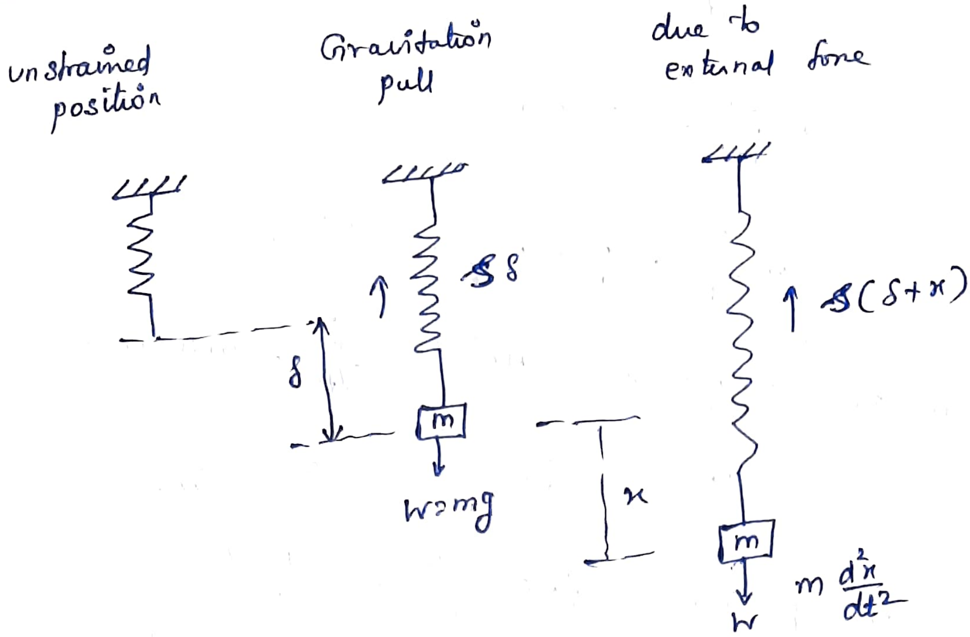
When the particles of the shaft (or) disc moves approximately perpendicular to the axis of the shaft, is known as transverse vibrations.

3. Torsional vibrations:-



When the particles of the shaft (or) disc move in a circle about the axis of the shaft is known as torsional vibrations.

Free longitudinal vibration



$mg = s\delta$

$$\boxed{\frac{s}{m} = \frac{g}{\delta}}$$

$s \rightarrow$ stiffness of the constraint (N/m)

$m \rightarrow$ mass of the body (kg)

$W \rightarrow$ weight of the body (N)

$\delta \rightarrow$ deflection (m)

$x \rightarrow$ displacement (m)

Restoring force $= W - s(\delta + x)$ $\quad (W = s\delta)$

$= \cancel{W} - \cancel{s\delta} - sx$

$= -sx$

$=$

Accelerating force $=$ Mass $\times$ acceleration

$= m \times \frac{d^2x}{dt^2}$

$$m \frac{d^2x}{dt^2} = -sx$$

$$m \frac{d^2x}{dt^2} + sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} x = 0 \longrightarrow (1)$$

Fundamental equation of SHM is $\frac{d^2x}{dt^2} + \omega^2 x = 0$ —— (2)

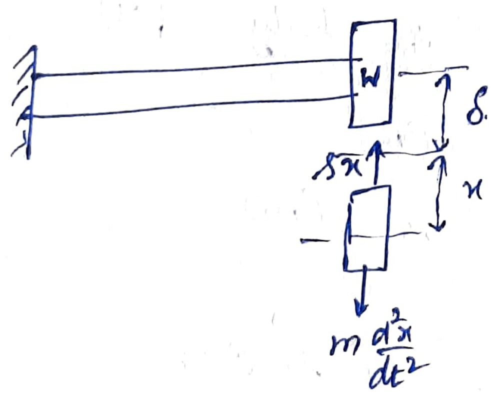
compare (1) & (2) $\omega^2 = \frac{s}{m} \Rightarrow \omega = \sqrt{\frac{s}{m}}$.

Time period, $t_p = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{s}}$

frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi} \sqrt{\frac{s}{m}}$

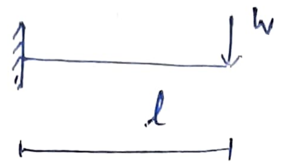
$$\boxed{\frac{1}{t_p} = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}.}$$

Natural frequency of free Transverse vibration.

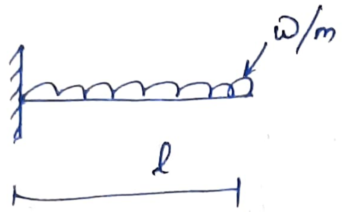


$$\boxed{f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}}.}$$

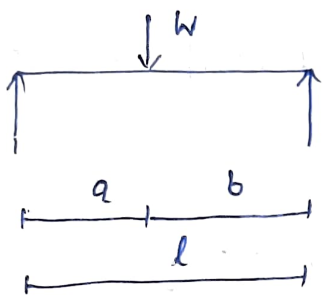
Transverse vibration

1. 

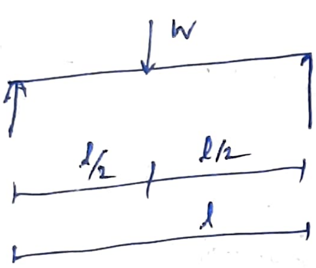
$$\delta = \frac{Wl^3}{3EI}$$

2. 

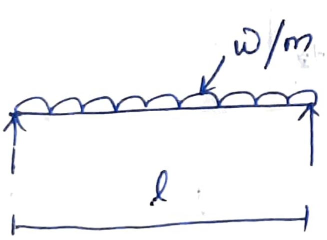
$$\delta = \frac{wl^4}{8EI}$$

3. 

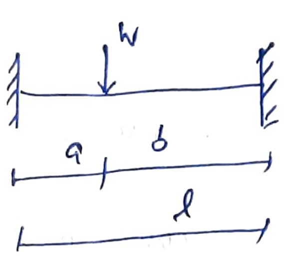
$$\delta = \frac{Wa^2b^2}{3EI\,l}$$

4. 

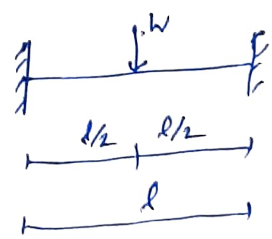
$$\delta = \frac{Wl^3}{48EI}$$

5. 

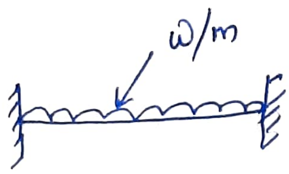
$$\delta = \frac{5}{384} \times \frac{wl^4}{EI}$$

6. 

$$\delta = \frac{Wa^3b^3}{3EI\,l^3}$$

7. 

$$\delta = \frac{Wl^3}{192EI}$$

8. 

$$\delta = \frac{wl^4}{384EI}$$

Longitudinal

$$\delta = \frac{Wl}{AE}.$$

$W \rightarrow$ Load, N.

$E \rightarrow$ young's Modulus, N/mm^2

$A \rightarrow$ cross section area, mm^2

$l \rightarrow$ length, mm.

① A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The young's modulus for the shaft material is 200 GN/m^2. Determine the frequency of longitudinal and transverse vibrations of the shaft.

Longitudinal,

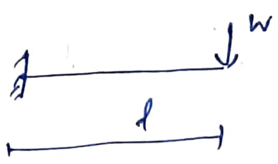
$$\delta = \frac{Wl}{AE} = \frac{100 \times 9.81 \times 300}{\frac{\pi}{4} \times 50^2 \times 200 \times 10^3}$$

$$\delta = 0.751 \times 10^{-3} \text{ mm} = 0.75 \times 10^{-6} \text{ m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.751 \times 10^{-6}}}$$

$$\boxed{f_n = 575 \text{ Hz}}$$

Transverse.



$$\delta = \frac{Wl^3}{3EI}$$

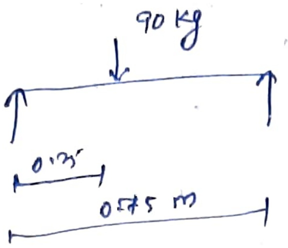
$$= \frac{100 \times 9.81 \times 300^3}{3 \times 200 \times 10^3 \times \frac{\pi}{64} \times 50^4} = 0.147 \text{ mm}$$

$$= 0.147 \times 10^{-3} \text{ m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$\boxed{f_n = 41 \text{ Hz}}$$

(2) A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibrations. Assume $E = 200\ GN/m^2$ and shaft diameter $= 50$ mm.

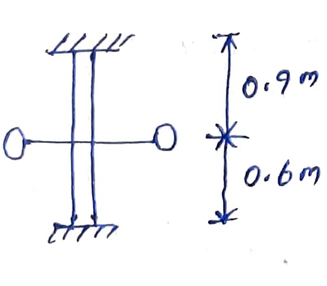


$$\delta = \frac{W a^2 b^2}{3EIl} = \frac{90 \times 9.81 \times 0.25^2 \times 0.5^2}{3 \times 200 \times 10^9 \times \frac{\pi}{64} \times 0.05^4 \times 0.75}$$

$$\delta = \underline{0.1 \times 10^{-3}\ m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \underline{49.85\ Hz}$$

(3) A flywheel is mounted on a vertical shaft as shown in fig. The both ends of the shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg. Find the natural frequency of tranverse vibration. Take $E = 200\ GN/m^2$



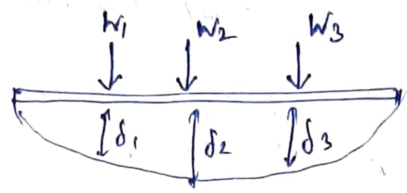
$$\delta = \frac{W a^3 b^3}{3EI l^3}$$

$$= \frac{500 \times 9.81 \times 0.9^3 \times 0.6^3}{3 \times 200 \times 10^9 \times \frac{\pi}{64} \times 0.05^4 \times 1.5^3}$$

$$\delta = \underline{1.24 \times 10^{-3}\ m}$$

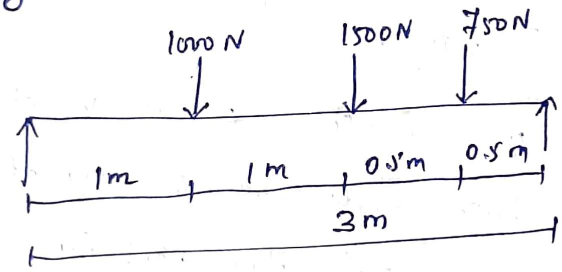
$$f_n = \frac{1}{2\pi}\sqrt{g/\delta} = \underline{14.24\ Hz}$$

Natural frequency of free transverse vibrations for a shaft subjected to a number of point loads.

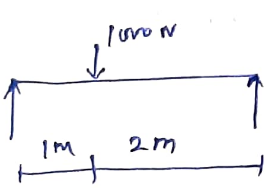


$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \Rightarrow \delta = \delta_1 + \delta_2 + \delta_3 .$$

① A shaft 50 mm diagram and 3 meters long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1m, 2m and 2.5 m from the left support. Take $E = 200\ GN/m^2$, find the frequency of transverse vibration.

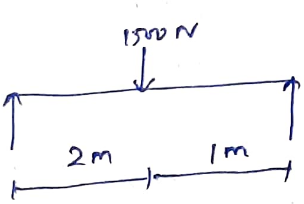


(i)



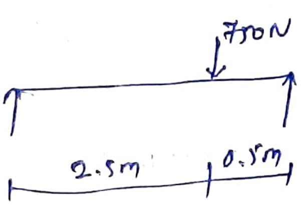
$$\delta_1 = \frac{wa^2b^2}{3EIl} = 7.24\times10^{-3}\ m.$$

(ii)



$$\delta_2 = 10.86\times10^{-3}\ m,$$

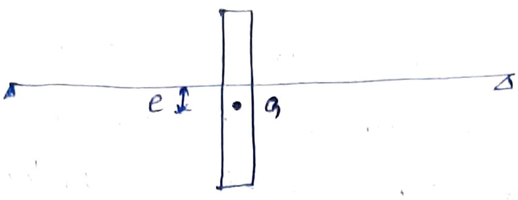
(iii)



$$\delta_3 = 2.12\times10^{-3}\ m.$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1+\delta_2+\delta_3}} = \underline{3.5\ Hz}$$

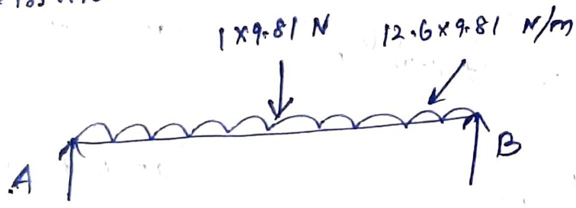
Critical (or) Whirling speed of a shaft.



* The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical (or) whirling speed.

* Critical (or) whirling speed is the same as the natural frequency of transverse vibration but its unit will be rps.

① Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid point. The density of the shaft material is 40×10^3 kg/m^3, and young's Modulus is 200 GN/m^2. Assume the shaft to be freely supported.



$m = 40 \times 10^3 \times \frac{\pi}{4} \times 0.02^2$

$= 12.6$ kg/m

$m = 12.6 \times 9.81$ N/m.

point load $\delta_1 = \frac{Wl^3}{48EI} = 28 \times 10^{-6}$ m

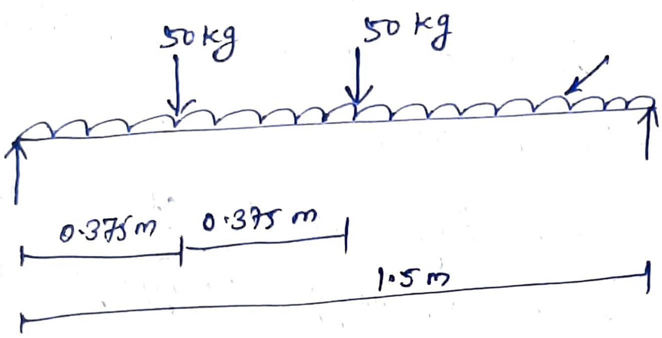
UDL, $\delta_2 = \frac{5wl^4}{384EI} = 0.133 \times 10^{-3}$ m

$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = 43.3$ Hz.

whirling speed, $= 43.3$ rps

$= 2598$ rpm

(2) A shaft 1.5 m long, supported in flexible bearings at the ends, carries two wheels each of 50 kg mass. One wheel is suitaated at the centre of the shaft and the other at a distance of 375 mm from the centre towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m³ and its modulus of elasticity is 200 GN/m². Find the whirling speed of the shaft, taking into account the mass of the shaft.



$$m = 7700 \times \frac{\pi}{4}\left[0.075^2 - 0.04^2\right] \times 1 = 24.34 \text{ kg/m}.$$

$$W = 24.34 \times 9.81 \text{ N/m}$$

(1) → $\delta_1 = \dfrac{Wa^2b^2}{3EIl} = 70 \times 10^{-6}$ m $(a = 0.375 \text{ \& } b = 1.125)$

(2) → $\delta_2 = 123 \times 10^{-6}$ m $(a = 0.75 \text{ \& } b = 0.75)$

(3) → $\delta_3 = \dfrac{5wl^4}{384EI} = 56 \times 10^{-6}$ m.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = 32.4 \text{ Hz}$$

Whirling speed = 32.4 rps
= 1944 rpm.

Damped free longitudinal vibration

→ Damping can be defined as the resistance offered by a body to the motion of a vibratory system.

→ The device used for this resisting purpose is called dampers.

→ It is usually assumed that the damping force is proportional to velocity across the damper.

Advantages of dampings

• The main advantage of providing damping in mechanical system is just to control the amplitude of vibration so that the failure due to resonance may be avoided.

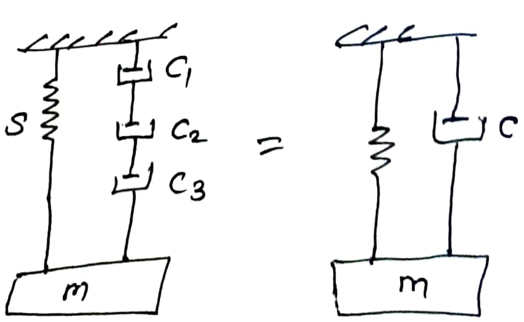
Damping Coefficient (c)

The damping force per unit velocity is known as damping coefficient.

$$c = \frac{\text{Damping force}}{\text{Velocity}}$$

$$\text{Damping force} = c\,\frac{dx}{dt}$$

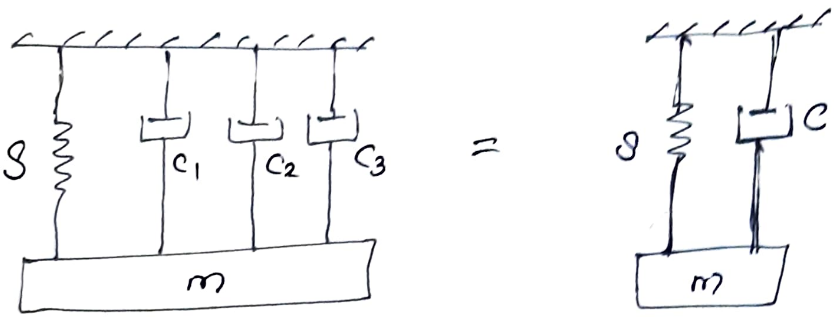
Dampers in series



The reciprocal of the effective damping coefficient of the dampers in series is the sum of the reciprocal of their individual damping coefficients.

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

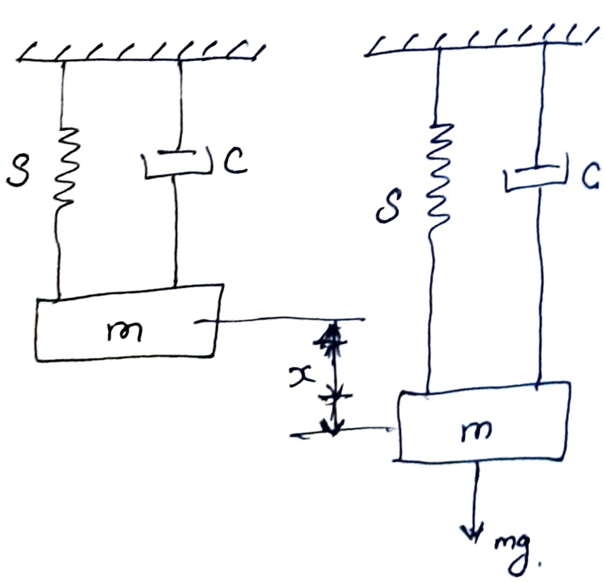
Dampers in parallel



The effective damping coefficient of the dampers in parallel is the sum of their individual damping coefficients.

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Frequency of free damped vibration.



Critical damping Coefficient,

$$\boxed{C_c = 2m\omega_n}$$

Damping Ratio :-

$$\xi = \frac{C}{C_c} = \frac{\text{Actual damping Coeff}}{\text{Critical damping Coeff}}$$

$$\boxed{\xi = \frac{C}{2m\omega_n}}$$

$\xi < 1 \rightarrow$ under damping system

$\xi = 1 \rightarrow$ Critical damping system

$\xi > 1 \rightarrow$ over damping system.

Frequency of free damped vibration $\Big\}$ $f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi}\sqrt{1-\xi^2}\cdot\omega_n$

$$\boxed{\omega_d = \sqrt{1-\xi^2}\cdot\omega_n}$$

Logarithmic decrement,

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad , \quad \delta = \ln\left[\frac{X_n}{X_{n+1}}\right]$$

system execute "n" cycles.

$$\delta = \frac{1}{n}\ln\left[\frac{X_0}{X_n}\right]$$

⊗ A vibrating system Consists of a mass of 8 kg, Spring of Stiffness 5·6 N/mm and a dashpot of damping Coefficient of 40 N/m/sec.
Find. (i) The critical damping Coefficient.
(ii) The damping factor
(iii) The natural frequency of damped vibration
(iv) The logarithmic decrement.
(v) The ratio of two Consecutive amplitudes
(vi) The number of cycles after which the original amplitude is reduced to 20 percent.

Sol

$m = 8\ kg$, $\quad S = 5\cdot6\ N/mm = 5\cdot6\times10^3\ N/m$

$C = 40\ N/m/sec.$

(i) $C_c = 2m\omega_n = 2m\sqrt{\frac{S}{m}} = 2\sqrt{Sm}$

$= 422\cdot32\ N/m/sec.$

(ii) Damping factor, $\xi = \frac{C}{C_c} = \frac{40}{422\cdot32} = 0\cdot0945$

(iii) Natural frequency of damped vibration $\Big\}$ $f_d = \frac{1}{2\pi}\omega_d$

$\omega_d = \sqrt{1-\xi^2}\ \omega_n = \sqrt{1-0\cdot0945^2}\cdot\sqrt{\frac{S}{m}}$

$\omega_d = 26\cdot34\ rad/sec.$

$$\boxed{f_d = 4\cdot173\ Hz}$$

(v) Logarithmic decrement

$$t_d = \frac{2\pi}{\omega_d}.$$

$$\delta = \xi \omega_n t_d$$

$$= \xi \omega_n \cdot \frac{2\pi}{\omega_d}.$$

$$\delta = \frac{2\pi \xi \times \cancel{\omega_n}}{\sqrt{1-\xi^2} \cdot \cancel{\omega_n}}$$

$$\boxed{\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}}}$$

$$\delta = 0.596$$

(vi) Ratio of two Consecutive amplitudes.

$$\delta = \ln \left[\frac{X_{n-1}}{X_n}\right]$$

$$0.596 = \ln \left[\frac{X_{n-1}}{X_n}\right]$$

$$e^{0.56} = \frac{X_{n-1}}{X_n}$$

$$\boxed{\frac{X_{n-1}}{X_n} = 1.8156}$$

$$\delta = \ln \left\{\frac{X_{n-1}}{X_n}\right\}$$

$$\frac{X_{n-1}}{X_n} = e^{\delta}$$

(vii) Number of cycles after $[X_n = (X_0 \times 0.2)]$

$$n\delta = \ln \left\{\frac{X_0}{X_n}\right\}$$

$$n [0.596] = \ln \left\{\frac{\cancel{X_0}}{0.2 \cancel{X_0}}\right\} = \ln [5].$$

$$\boxed{n = 2.7 \text{ cycles}}$$

(2) An instrument vibrates with a frequency of 1.24 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 1.03 Hz. Find
(i) Damping factor (ii) The logarithmic decrement.

GD $f_n = \frac{1}{2\pi}\omega_n = 1.24$ Hz

$f_d = \frac{1}{2\pi}\omega_d = 1.03$ Hz.

Sol

$\omega_n = 1.24 \times 2 \times \pi = 7.791$ rad/sec

$\omega_d = 1.03 \times 2 \times \pi = 6.472$ rad/sec

$\omega_d = \sqrt{1-\xi^2}\,\omega_n$

$6.472^2 = [1-\xi^2] \times 7.791^2$

$\boxed{\xi = 0.556}$ Damping factor.

Logarithmic decrement (δ)

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.556}{\sqrt{1-0.556^2}}$$

$\delta = 4.21$

(3) A mass suspended from a helical spring vibrates in a viscous fluid medium whose resistance varies directly with the speed. It is observed that the frequency of damped vibration is 90 per min and the amplitude decreases to 20% of its initial value in one complete vibration. Find the frequency of the free undamped vibration of the system.

$$f_d = 90/\text{min} = 1.5 \text{ Hz.}$$

$$X_1 = 0.2 X_0$$

$$\frac{X_0}{X_1} = \frac{1}{0.2} = 5$$

$$\delta = \ln\left[\frac{X_0}{X_1}\right] = \ln[5] = \underline{\underline{1.609}}$$

$$\therefore \delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$1.609 = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = \underline{\underline{0.247}}$$

$$f_d = \frac{1}{2\pi}\omega_d \Rightarrow 1.5 = \frac{1}{2\pi} \times \omega_d$$

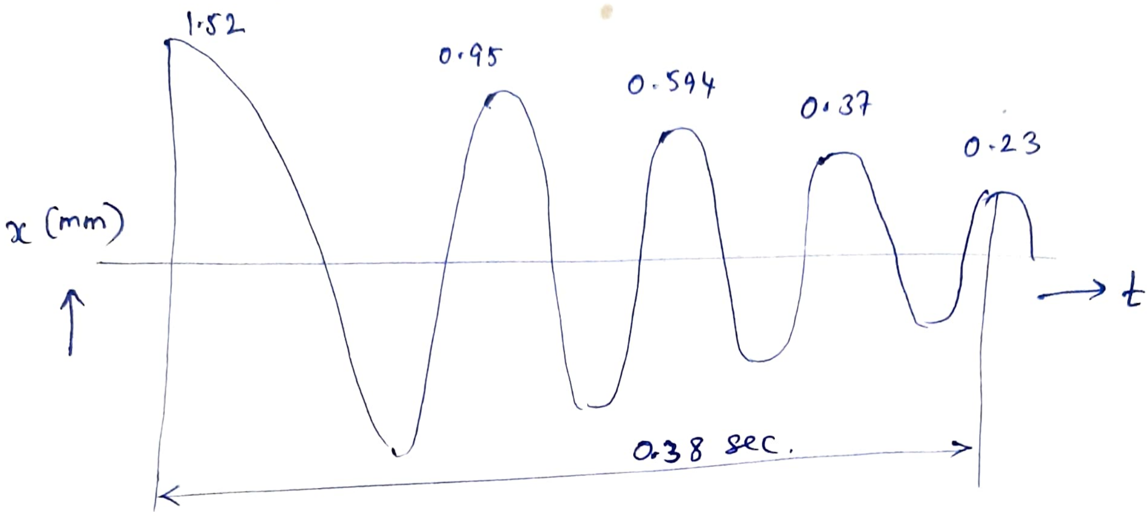
$$\omega_d = \underline{\underline{9.42}} \text{ rad/sec}$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n.$$

$$\omega_n = \underline{\underline{9.72}} \text{ rad/sec}$$

$$\therefore f_n = \frac{1}{2\pi}\omega_n = \underline{\underline{1.548 \text{ Hz.}}}$$

(4) The machine has a mass of 200 kg. It is placed on two different isolators and the corresponding free vibration record is shown in Fig. Determine (i) damping factor (ii) natural frequency of damped vibration (iii) natural frequency of undamped vibration, (iv) spring stiffness (v) Critical damping coefficient, and (vi) damping Coefficient.



$m = 200 kg$, $X_0 = 1.52$, $X_1 = 0.95$, $X_2 = 0.594$, $X_3 = 0.34$, $X_4 = 0.23$

$$\frac{X_0}{X_1} = \frac{X_1}{X_2} = \cdots = 1.6.$$

$$\delta = \ln\left[\frac{X_0}{X_1}\right] \Rightarrow \delta = \ln(1.6) = \underline{\underline{0.47}}$$

(i) $$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = \underline{\underline{0.0744}}$$

(ii) f_d = No of cycles per sec

$= 4/0.38$

$= \underline{10.526 \text{ Hz}}$

(iii) $$f_d = \frac{1}{2\pi}\omega_d.$$

$$10.526 = \frac{1}{2\pi}\omega_d$$

$$\omega_d = 66.14 \text{ rad/sec}$$

(iii) $f_n = \frac{1}{2\pi} \omega_n$

$f_n = \frac{1}{2\pi} \times 66.32$

$\boxed{f_n = 10.56 \text{ Hz}}$

$\omega_d = \sqrt{1-\xi^2}\, \omega_n$

$\omega_n = 66.32$ rad/sec

(iv) spring stiffness (S)

$\omega_n = \sqrt{\frac{S}{m}} \rightarrow S = \omega_n^2 \times m$

$= 879668$ N/m

~~879.668 N/mm~~

(v) Critical damping Coefficient (Cc)

$C_c = 2m\omega_n = 2 \times 200 \times 66.32$

$= 26528$ N/m/sec

(vi) damping Coefficient (C)

(1) A body of mass 50 kg is supported by an elastic structure of stiffness 10 kN/m. The motion of the body is controlled by a dashpot such that the amplitude of vibration decreases to one tenth of its original value after two complete vibrations. Determine

(i) the damping ratio

(ii) The damping force at 1 m/sec

(iii) the natural frequency of damped vibration.

$m = 50$ kg.

$S = 10$ kN/m $= 10 \times 10^3$ N/m.

$n = 2$

$X_2 = \frac{1}{10} X_0$, $v = 1$ m/sec

$$n\delta = \ln\left[\frac{X_0}{X_n}\right] \Rightarrow 2\delta = \ln\left[\frac{X_0}{\frac{1}{10} \times X_0}\right]$$

$$\delta = 1.151$$

$$\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = 0.18$$

$$\xi = \frac{c}{C_c}$$

$$C_c = 2 m \omega_n = 2m\sqrt{\frac{S}{m}} = 2 \times \cancel{10 \times 10^3} \cdot 50 \times \sqrt{\frac{10 \times 10^3}{50}} = $$

$$c = C_c \xi$$

$$c = 254.56 \text{ N/m/sec}$$

$$c = \frac{\text{Damping force}}{\text{velocity}}$$

$$\text{Damping force} = c \times \text{velocity} = 254.56 \times 1 = 254.56 \text{ N}$$

Natural frequency of damped vibration

$$f_d = \frac{1}{2\pi} \omega_d$$

$$\omega_d = \sqrt{1-\xi^2}\ \omega_n = 13.91 \text{ rad/sec}$$

$$f_d = \frac{1}{2\pi} \times 13.91 = \underline{\underline{2.214 \text{ Hz}}}$$

(2) In a single degree damped vibrating system, the suspended mass of 3.75 kg makes 12 oscillations in 7 seconds when disturbed from its equillibrium position. The amplitude decreases to 0.33 of the initial value after 4 oscillations. Determine.

(i) The stiffness of the spring.
(ii) The logarithmic decrement
(iii) The damping factor
(iv) damping coefficient

$m = 3.75$ kg, $N = 12$, $t = 7$ sec, $X_4 = 0.33 X_0$

$$f_n = \frac{12}{7} = \underline{\underline{1.71 \text{ Hz}}}$$

$$f_n = \frac{1}{2\pi} \omega_n \Rightarrow \omega_n = 10.77 \text{ rad/sec.}$$

(i) $\omega_n = \sqrt{\frac{s}{m}} \Rightarrow s = \underline{\underline{435.07}}$ N/m

(ii) $n\delta = \ln\left[\frac{X_0}{X_n}\right] \Rightarrow 4\delta = \ln\left[\frac{X_0}{X_4}\right]$

$$4\delta = \ln\left[\frac{X_0}{0.33X_0}\right]$$

$$\boxed{\delta = 0.277}$$

$$\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = 0.044$$

(iv) damping coefficient, c

$$\xi = \frac{c}{c_c} \qquad c_c = 2 m \omega_n =$$

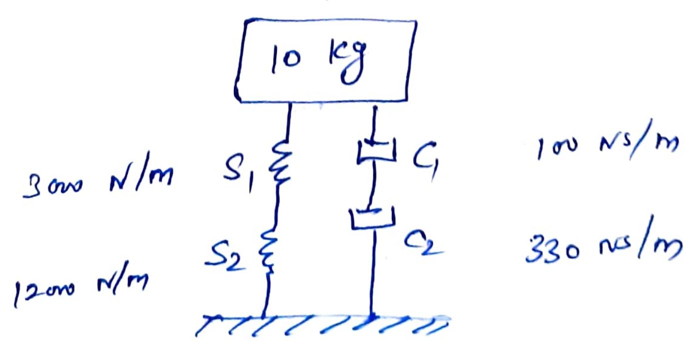
$$c = \xi c_c$$

$$= 0.044 \times 2 \times 3.75 \times 10.77.$$

$$\boxed{c = 3.566 \text{ N/m/sec}}$$

(3) Between a solid mass of 10 kg and the floor are kept two slaps of isolators, natural rubber and felt in series. The natural rubber slap has a stiffness of 3000 N/m and an equivalent viscous damping coefficient of 100 Nsec/m. The felt has a stiffness of 12000 N/m and equivalent viscous damping coefficient of 330 Nsec/m. Determine the undamped and the damped natural frequencies of the system.

m = 10 kg.



$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} \Rightarrow S_{eq} = 2400 \ \text{N/m}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = 76.744 \ \text{Ns/m}.$$

Natural frequency of undamped system

$$f_n = \frac{1}{2\pi}\omega_n \qquad \omega_n = \sqrt{\frac{S}{m}}$$

$$= \sqrt{\frac{2400}{10}}$$

$$f_n = 2.465 \ \text{Hz}$$

$$= 15.49 \ \text{rad/sec}$$

Natural frequency of damped system

$$f_n = \frac{1}{2\pi}\omega_d \qquad \omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$\xi = \frac{C}{C_c}$$

$$\cancel{\xi = \frac{C}{C_c}}$$

$$\cancel{C = \xi C_c}$$

$$\xi = \frac{76.744}{2 m \omega_n}$$

$$= \frac{76.744}{2 \times 10 \times 15.49}$$

$$= 0.247$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$= 15 \ \text{rad/sec}$$

$$f_n = \frac{1}{2\pi} \times 15$$

$$f_d = 2.388 \ \text{Hz}$$