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Course material
Business Mathematics
BC404



Sets and operations



Sets and operations

- . Basic concepts
 - ❖ Definitions of sets and operations
 - ❖ Addition and subtraction of sets
 - ❖ Problems using set addition and subtraction

Aim:

To construct strong knowledge in
sets and its operations



Objectives:

- ❖ To expose students to important mathematical concepts and tools that can be applied to solve practical business problems
- ❖ To understand the basic concepts of Mathematics
- ❖ To have proper understanding of mathematical applications in economics , Finance,, Commerce and Management

Prerequisites:

Sets, operations, matrices, inverse of matrices, adjoint, cofactor, differentiation, Integration, mathematical symbols and notations and some formulas

Co-requisites:

Mathematical symbols and notations and few formulas



Learning outcomes:

- ❖ Connect acquired knowledge and skills with practical problems in economic practice.
- ❖ Discern effects of various types and methods of interest accounts

Course outcomes:

- ❖ Define basic terms in the areas of business mathematics
- ❖ Explain the concepts and use equations, formulae, and mathematical expressions and relationships in a variety of contexts
- ❖ Apply the knowledge in mathematics (sets, matrices, integration, differentiation) in solving business problems
- ❖ Analyze and demonstrate mathematical skills required in mathematically intensive areas in Economics and business
- ❖ Evaluate problems in the areas of business mathematics, simple and compound interest account and consumer credit

Sets and operations

What is the meaning of sets in mathematics?

A set in mathematics is a collection of well -defined and distinct objects. Sets are one of the most fundamental concepts in mathematics

Why we study sets in mathematics?

The purpose of sets is to house a collection of related objects. They are important everywhere in mathematics uses or refer to sets in some way. They are important for building more complex mathematical structure,



Objects form a set

How to state that whether the objects form a set or not?

- ❖ A collection of ‘lovely flowers’ is not a set, because the objects (flowers) to be included are not well-defined.
Reason: The word “lovely” is a relative term. What may appear lovely to one person may not be so to the other person.
- ❖ A collection of “Yellow flowers” is a set, because every red flowers will be included in this set i.e., the objects of the set are well-defined.
- ❖ A group of “Young singers” is not a set, as the range of the ages of young singers is not given and so it can’t be decided that which singer is to be considered young i.e., the objects are not well-defined

Elements of a Set

What are the elements of a set or members of a set?

The objects used to form a set are called its element or its members.

Generally, the elements of a set are written inside a pair of curly (idle) braces and are represented by commas. The name of the set is always written in capital letter.

Solved Examples to find the elements or members of set:

1. $A = \{v, w, x, y, z\}$

Here 'A' is the name of the set whose elements (members) are v, w, x, y, z.

2. If a set $A = \{3, 6, 9, 10, 13, 18\}$. State whether the following statements are 'true' or 'false':

(i) $7 \in A$

(ii) $12 \notin A$

(iii) $13 \in A$

(iv) $9, 12 \in A$

(v) $12, 14, 15 \in A$

Solution:

(i) $7 \in A$

False, since the element 7 does not belong to the given set A.

(ii) $10 \notin A$

False, since the element 10 belongs to the given set A.

(iii) $13 \in A$

True, since the element 13 belongs to the given set A.

(iv) $9, 10 \in A$

True, since the elements 9 and 12 both belong to the given set A.

(v) $10, 13, 14 \in A$

False, since the element 14 does not belong to the given set A.



Properties of Sets

What are the two basic properties of sets?

The two basic properties to represent a set are explained below using various examples.

1. The change in order of writing the elements does not make any changes in the set.

In other words the order in which the elements of a set are written is not important. Thus, the set $\{a, b, c\}$ can also be written as $\{a, c, b\}$ or $\{b, c, a\}$ or $\{b, a, c\}$ or $\{c, a, b\}$ or $\{c, b, a\}$.

For Example:

Set $A = \{4, 6, 7, 8, 9\}$ is same as set $A = \{8, 4, 9, 7, 6\}$

i.e., $\{4, 6, 7, 8, 9\} = \{8, 4, 9, 7, 6\}$

Similarly, $\{w, x, y, z\} = \{x, z, w, y\} = \{z, w, x, y\}$ and so on

2. If one or many elements of a set are repeated, the set remains the same.

In other words the elements of a set should be distinct. So, if any element of a set is repeated number of times in the set, we consider it as a single element. Thus, $\{1, 1, 2, 2, 3, 3, 4, 4, 4\} = \{1, 2, 3, 4\}$

The set of letters in the word 'GOOGLE' = $\{G, O, L, E\}$

For Example:

The set $A = \{5, 6, 7, 6, 8, 5, 9\}$ is same as set $A = \{5, 6, 7, 8, 9\}$

i.e., $\{5, 6, 7, 6, 8, 5, 9\} = \{5, 6, 7, 8, 9\}$

In general, the elements of a set are not repeated


OPERATIONS ON SETS

Definition of operations on sets:

When two or more sets combine together to form one set under the given conditions, then operations on sets are carried out

What are the four basic operations on sets?

The four basic operations are:

1. Union of Sets
 2. Intersection of sets
 3. Complement of the Set
 4. Cartesian Product of sets
- 

Union of Sets

Union of two given sets is the smallest set which contains all the elements of both the sets.

To find the union of two given sets A and B is a set which consists of all the elements of A and all the elements of B such that no element is repeated.

The symbol for denoting union of sets is 'U'.

For example;

Let set $A = \{2, 4, 5, 6\}$ and set $B = \{4, 6, 7, 8\}$

Taking every element of both the sets A and B, without repeating any element, we get a new set = $\{2, 4, 5, 6, 7, 8\}$

Therefore, symbolically, we write union of the two sets A and B is $A \cup B$ which means A union B.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

INTERSECTION OF SETS

Definition of Intersection of Sets:

Intersection of two given sets is the largest set which contains all the elements that are common to both the sets.

To find the intersection of two given sets A and B is a set which

consists of all the elements which are common to both A and B.

For example:

Let set A = {2, 3, 4, 5, 6} and set B = {3, 5, 7, 9}

In this two sets, the elements 3 and 5 are common. The set containing these common elements i.e., {3, 5} is the intersection of set A and B symbol for denoting intersection of sets is ' \cap '

$$\mathbf{A \cap B = \{x : x \in A \text{ and } x \in B\}}$$

PROBLEMS

1. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Solution:

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

then
$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 20 + 28 - 36$$

$$= 48 - 36$$

$$= 12$$

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

Solution:

Using the formula $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$$70 = 18 + 25 + n(B - A)$$

$$70 = 43 + n(B - A)$$

$$n(B - A) = 70 - 43$$

$$n(B - A) = 27$$

$$\text{Now } n(B) = n(A \cap B) + n(B - A)$$

$$= 25 + 27$$

$$= 52$$

3. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

Solution:

Let A = Set of people who like cold drinks.

B = Set of people who like hot drinks.

Given

$$(A \cup B) = 60 \quad n(A) = 27 \quad n(B) = 42$$

then;

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 27 + 42 - 60$$

$$= 69 - 60 = 9$$

$$= 9$$

Therefore, 9 people like both tea and coffee.

4. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

(i) When two classes meet at different hours and 12 students are enrolled in both activities

(ii) When two classes meet at the same hour.

Solution:

$$n(A) = 35, \quad n(B) = 57, \quad n(A \cap B) = 12$$

Let A be the set of students in art class.

B be the set of students in dance class.

(i) When 2 classes meet at different hours

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 35 + 57 - 12 \\ &= 92 - 12 \\ &= 80 \end{aligned}$$

(ii) When two classes meet at the same hour,

$$A \cap B = \emptyset \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= n(A) + n(B)$$

$$= 35 + 57$$

$$= 92$$

5. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Solution:

Let A be the set of people who speak English.

B be the set of people who speak French.

$A - B$ be the set of people who speak English and not French.

$B - A$ be the set of people who speak French and not English.

$A \cap B$ be the set of people who speak both French and English.

Given,

$$n(A) = 72 \quad n(B) = 43 \quad n(A \cup B) = 100$$

$$\text{Now, } n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 72 + 43 - 100$$

$$= 115 - 100$$

$$= 15$$

Therefore, Number of persons who speak both French and English = 15

$$n(A) = n(A - B) + n(A \cap B)$$

$$\Rightarrow n(A - B) = n(A) - n(A \cap B)$$

$$= 72 - 15$$

$$= 57$$

and $n(B - A) = n(B) - n(A \cap B)$

$$= 43 - 15$$

$$= 28$$

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28

Problems for practice:

1. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?
2. . Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play (i) chess and carrom. (ii) chess, carrom but not scrabble

3. If $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$ find a) $A \cup B$ b) $A \cap B$ c) $A - B$

4. If $A = \{0, 1, 3, 5\}$ $B = \{1, 2, 4, 7\}$ $C = \{1, 2, 3, 5, 8\}$

find $(A \cap C) \cup (B \cap C)$.

5. $U = \{1, 2, 3, 4, 5, 6, 7\}$ $A = \{2, 3, 4\}$, $B = \{2, 3, 4\}$.

Verify that $(A \cap B)' = A' \cup B'$

6. $U = \{1, 2, 3, 4, 5, 6, 7\}$ $A = \{2, 3, 4\}$, $B = \{2, 3, 4\}$.

Verify that $A - B = A \cap B' = B' - A'$

7. In a city there are three newspapers A, B and C published. 42% of the people in that city read A, 51% read B, 68% read C, 30% read both A and B, 28% read B and C, 36% read C and A, 8% do not read any of the newspaper. Find the % of read all the newspapers

Multiple choice questions with answers:

1. A _____ is an ordered collection of objects.

- a) Relation
- b) Function
- c) Set
- d) Proposition

Answer: c

Explanation: By the definition of set.

2. $A = \{1, 3, 5, 7, 9\}$ is a _____.

- A. null set
- B. finite set
- C. singleton set
- D. infinite set

ANSWER: B

3. The number of Indians in the world is _____.

- A. finite set
- B. universal set
- C. infinite set
- D. equal set

ANSWER: A

4. What is the cardinality of the set of odd positive integers less than 10?

- a) 10
- b) 5
- c) 3
- d) 20

Answer: b

Explanation: Set S of odd positive an odd integer less than 10 is {1, 3, 5, 7, 9}. Then, Cardinality of set S = |S| which is 5.

5. The union of the sets $\{1, 2, 5\}$ and $\{1, 2, 6\}$ is the set

a) $\{1, 2, 6, 1\}$

b) $\{1, 2, 5, 6\}$

c) $\{1, 2, 1, 2\}$

d) $\{1, 5, 6, 3\}$

Answer: b

Explanation: The union of the sets A and B, is the set that contains those elements that are either in A or in B

6. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then the number of elements in $A \cup B$ is?

- a) 4
- b) 5
- c) 6
- d) 7

Answer: a

Explanation: $A \cup B$ is $\{1, 2, 3, 4\}$

7. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then number of elements in $A \cap B$ is?

- a) 1
- b) 2
- c) 3
- d) 4

Answer: b

Explanation: $A \cap B$ is $\{2, 3\}$.


8. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then number of elements in $A \cap B$ is?
- a) 1
 - b) 2
 - c) 3
 - d) 4

Answer: b

Explanation: $A \cap B$ is $\{2, 3\}$

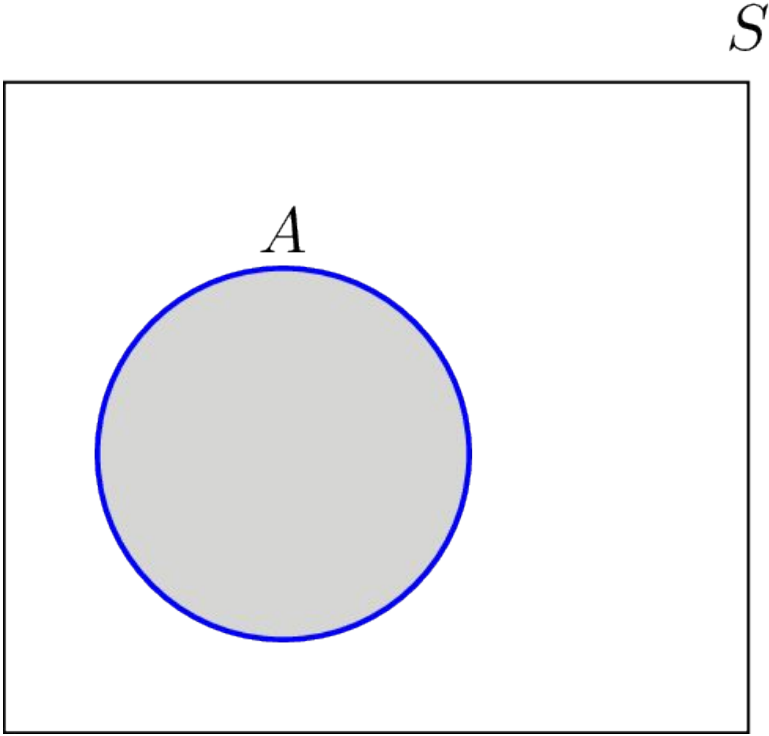


Sets - Venn diagrams

- ❖ Basic concepts
 - ❖ Definitions of Venn diagram
 - ❖ Venn diagram incase of two elements
 - ❖ Addition and subtraction using Venn diagram
 - ❖ Problems using Venn diagram
- 

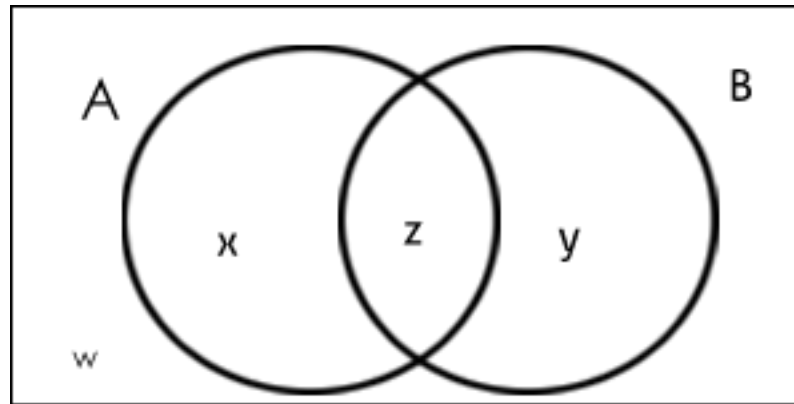
Venn Diagrams

Venn diagrams are very useful in visualizing relation between sets. In a Venn diagram any set is depicted by a closed region. shows an example of a Venn diagram given below. In this figure, the big rectangle shows the universal set S . The shaded area shows another set A



Venn Diagram

Venn Diagram in case of two elements



Where;

X = number of elements that belong to set A only

Y = number of elements that belong to set B only

Z = number of elements that belong to set A and B both
($A \cap B$)

W = number of elements that belong to none of the sets
A or B

From the above figure, it is clear that

$$n(A) = x + z ;$$

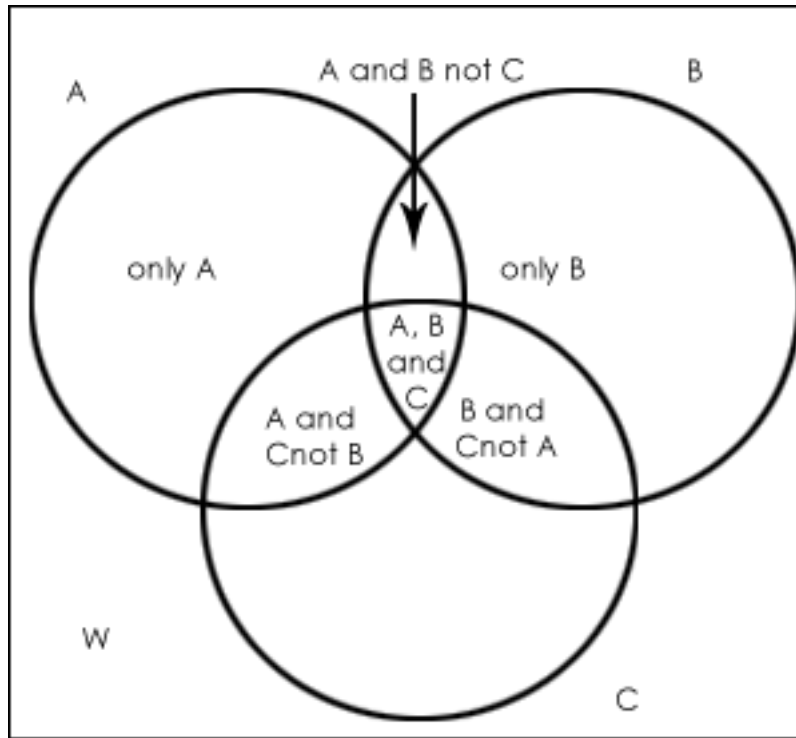
$$n(B) = y + z ;$$

$$n(A \cap B) = z ;$$

$$n(A \cup B) = x + y + z .$$

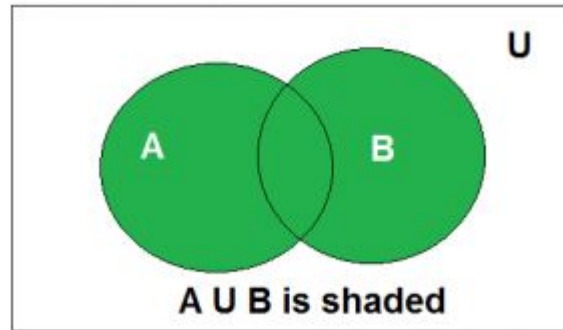
$$\text{Total number of elements} = x + y + z + w$$

Venn Diagram in case of three elements



Union

Union of the sets A and B , denoted by $A \cup B$, is the set of distinct element belongs to set A or set B , or both.

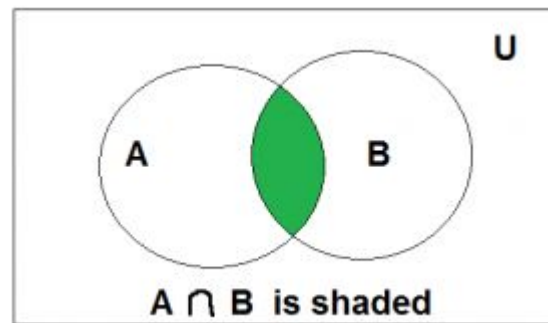


Venn Diagram of $A \cup B$

Example : Find the union of $A = \{2, 3, 4\}$ and $B = \{3, 4, 5\}$;

Intersection

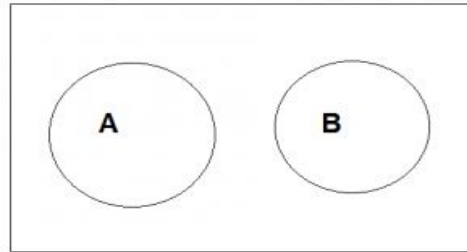
The intersection of the sets A and B , denoted by $A \cap B$, is the set of elements belongs to both A and B i.e. set of the common element in A and B .



Venn Diagram of $A \cap B$

Disjoint

Two sets are said to be disjoint if their intersection is the empty set .i.e sets have no common elements.



Venn Diagram of A disjoint
B

For Example

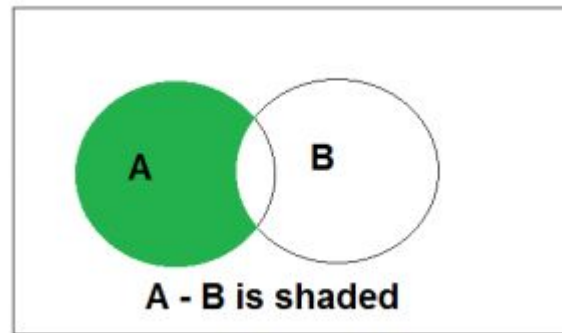
Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$.

A and B are disjoint set both of them have no common elements

Set Difference

Difference between sets is denoted by $A - B$, is the set containing elements of set A but not in B .

i.e all elements of A except the element of B



Venn Diagram of
 $A - B$.

Example 1:

1. In a college, 200 students are randomly selected. 140 like tea, 120 like coffee and 80 like both tea and coffee.

How many students like only tea?

How many students like only coffee?

How many students like neither tea nor coffee?

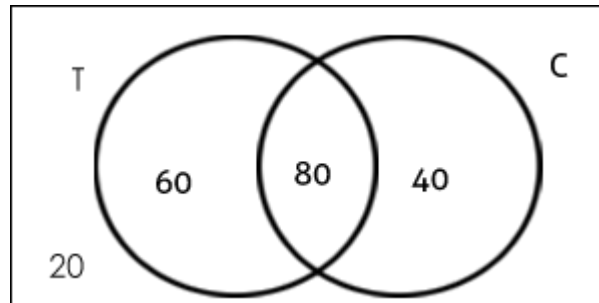
How many students like only one of tea or coffee?

How many students like at least one of the beverages?



Solution:

The given information may be represented by the following Venn diagram, where T = tea and C = coffee.



Number of students who like only tea = 60

Number of students who like only coffee = 40


Number of students who like neither tea nor coffee = 20

Number of students who like only one of tea or coffee

$$= 60 + 40 = 100$$


Number of students who like at least one of tea or coffee =

n (only Tea) + n (only coffee) + n (both Tea & coffee)

$$= 60 + 40 + 80 = 180$$


Example 2:

In a survey of 500 students of a college, it was found that 49% liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both and 28% liked watching football and basket ball both. 5% liked watching none of these games

- * How many students like watching all the three games?
 - * Find the ratio of number of students who like watching only football to those who like watching only hockey.
 - * Find the number of students who like watching only one of the three given games.
 - * Find the number of students who like watching at least two of the given games
- 

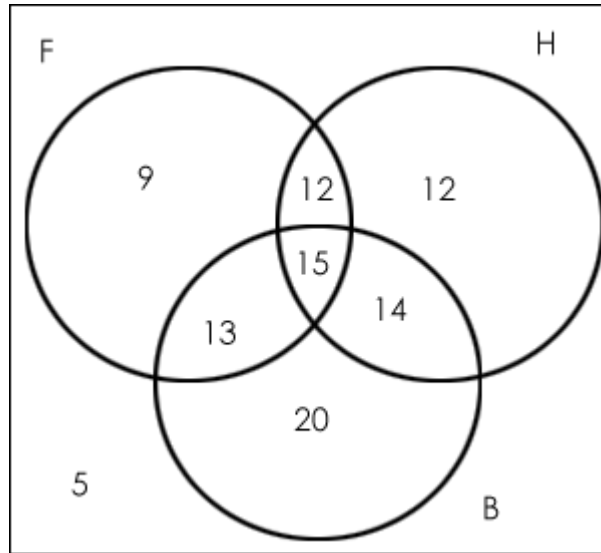
Solution:

$n(F)$ = percentage of students who like watching football
= 49%

$n(H)$ = percentage of students who like watching hockey
= 53%

$n(B)$ = percentage of students who like watching
basketball = 62%

$n(F \cap H) = 27\%$; $n(B \cap H) = 29\%$; $n(F \cap B) = 28\%$



Number of students who like watching all the three games
= 15 % of 500 = 75.

Ratio of the number of students who like only football to
those who like only hockey
= (9% of 500)/(12% of 500) = 9/12 = 3:4.

The number of students who like watching only one of the
three given games
= (9% + 12% + 20%) of 500 = 205

The number of students who like watching at least two of
the given games=(number of students who like watching
only two of the games) +(number of students who like
watching all the three games)= (12 + 13 + 14 + 15)% i.e.
54% of 500 = 270.

Assignment problems:

1. $U = \{1,2,3,4,5,6,7\}$ $A = \{2,3,4\}$, $B = \{2,3,4\}$.

Verify that $(A \cap B)' = A' \cup B'$


2. $U = \{1,2,3,4,5,6,7\}$ $A = \{2,3,4\}$, $B = \{2,3,4\}$.

Verify that $A - B = A \cap B' = B' - A'$

3. Draw Venn diagram for Associative law.

4. Draw Venn diagram for Distributive law.

5. Use Venn diagrams to verify $(A \cap B)' = A' \cup B'$



Reference Books

1. An Introduction to Business Mathematics - V. Sundaresan & S.D. Jeyaseelan, 6th Edition
2008, S. Chand & Company Ltd, Ram Nagar, New Delhi - 110 055.
2. Business Mathematics - Kashyap Trivedi & Chirag Trivedi New Edition 2011, Dorling Kindersley (I) P Ltd 11, Community Centre Panchasheel Park New Delhi – 11 017.

Links for reference

1. <http://youtu.be/4jE7wSY6q0Y>
2. <https://youtu.be/WSS5EiJ2gew>

Thank you

