LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

What are linear differential equations with constant coefficients?

We learn about Homogeneous linear differential equations Complimentary functions
Auxilliary equations

We learn how to calculate the solution when then the roots are distinct, coincident and complex.

Presenting the easiest way to learn Mathematics!

A differential equation of the form

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}}{dx^{n-1}} y + \text{---} - a_n y = f(x)$$

is said to be a linear differential equation with constant coefficients

$$D^n y + a_1 D^{n-1} y + \text{-------} - a_n y = f(x) \quad \text{where } D^n = \frac{d^n}{dx^n}$$

If f(x) = 0, then $D^n y + a_1 D^{n-1} y + \text{-----} - a_n y = 0$

is called a homogenous linear differential equation

Given $D^2 + a_1 D + a_2 = 0$

If y_1, y_2 *are solutions of the above equation, then*

$c_1 y_1 + c_2 y_2 = y_c$ is called the general solution

$If\ we\ replace\ m\ by\ D,\ m^2 + a_1 m + a_2 = 0$

This is called the auxillary equation

Let m_1, m_2 *be distinct real roots of the auxillary equation*

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$ *is the general solution*

If the roots are real and coincident, say, $m_1 = m_2 = m$ *then*

$$y = (c_1 + c_2 x)e^{mx} \text{ is the general solution}$$

Let the roots be complex, say $p \pm iq$, *then* $y = e^{px}(c_1 \cos qx + c_2 \sin qx)$

is the general solution

Q1

Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$

Auxillary equation

$4m^4 - 8m^3 - 7m^2 + 11m + 6 = 0$

m = -1 is a root

-1	4	-8	-7	11	6
		-4	12	-5	-6
	4	-12	5	6	0

(m+1)$(4m^3 - 12m^2 + 5m + 6) = 0$

Now consider $4m^3 - 12m^2 + 5m + 6 = 0$

2 is a root of the equation

2	4	-12	5	6
		8	-8	-6
	4	-4	-3	0

$$(m+1)(m-2)(4m^2-4m-3)=0$$

$$(m+1)(m-2)(2m(2m-3)+1(2m-3))=0$$

$$(m+1)(m-2)(2m-3)(2m+1)=0$$

Roots are -1,2, $^{3}/_{2}$, $-^{1}/_{2}$

Solution is $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{^{3}/_{2}x} + c_4 e^{-^{1}/_{2}x}$

The best way to learn Mathematics is to do Mathematics.

Practice these problems once or twice by yourself while learning.

Q 2

$Solve\ (D^4-1)y = 0$

Auxillary equation is

$$(m^4 - 1) = 0$$

$$(m^2 + 1)(m^2 - 1) = 0$$

$$m = \pm 1,\ m^2 = -1$$

Roots are $\pm 1, \pm i$

Solution is $y_c = c_1 e^x + c_2 e^{-x} + c_3 cosx + c_4 sinx$

Finding the general solution of a homogeneous equation

when The roots are real and distinct

The roots are real and

coincident The roots are

complex